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Experimental quantum simulations: towards exotic quantum many-body physics with nuclear spins

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Frontier Lecture Sunday, 21th Oct. from 9:00 to 10:00

Outline

I. Motivation

- Quantum simulation: Why? What?
- Spin-based Quantum information processing

II. NMR quantum simulators

- Our platform & basic principles

III. Recent experimental QS results

- Detecting topological quantum phase transition
- Identifying Z2 topological order and its breakdown
- Measuring out-of-time correlation
- Controlling quantum dynamics based on QS

IV. Conclusion and Outlook

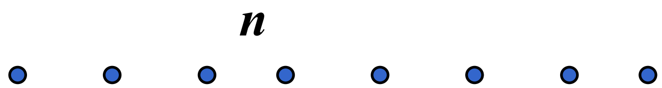
Quantum simulation: Why?

Simulating of quantum systems with computers

- **Classical computers**

Exponential growth of Hilbert space

Computational basis



$$|\Psi\rangle = \sum_{i_1=1}^{2} \cdots \sum_{i_n=1}^{2} c_{i_1 \ldots i_n} \, | \, i_1 \ldots i_n \rangle$$

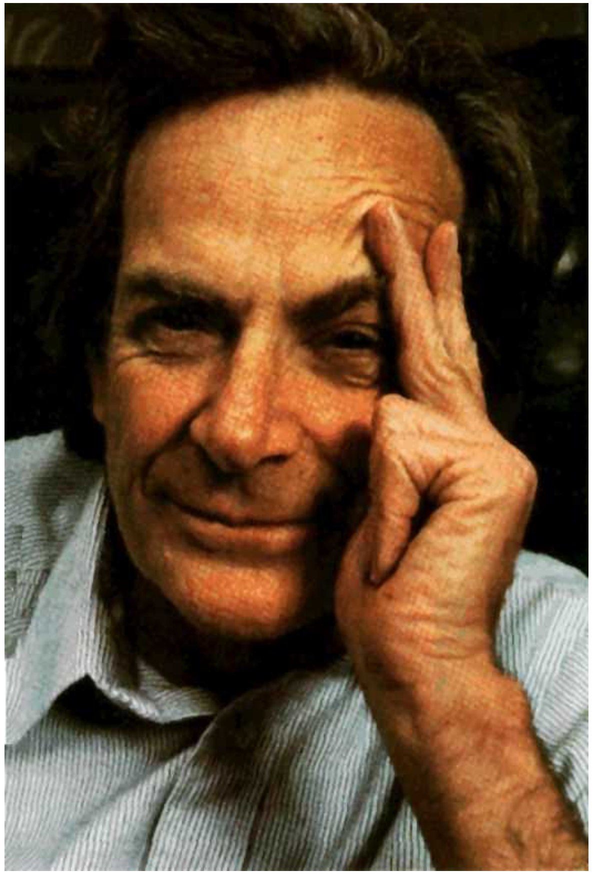
System with 50 qubits → $2^{50} \approx 10^{15}$ complex amplitudes ~ 32 x 10^{15} bytes of information → well beyond the capacity of existing computers

The Puzzle: Feynman's main thesis was quantum systems could not be efficiently imitated on classical systems.

Quantum simulation: What?

Controllable Quantum Systems As Simulators

- **Quantum computers - Universal quantum simulators**

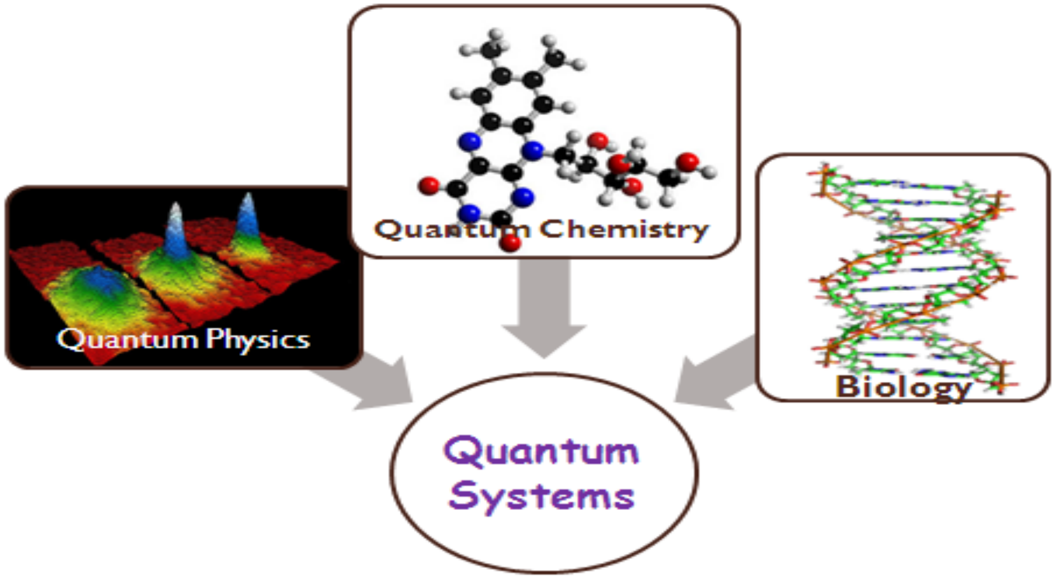


1982 Richard P. Feynmann

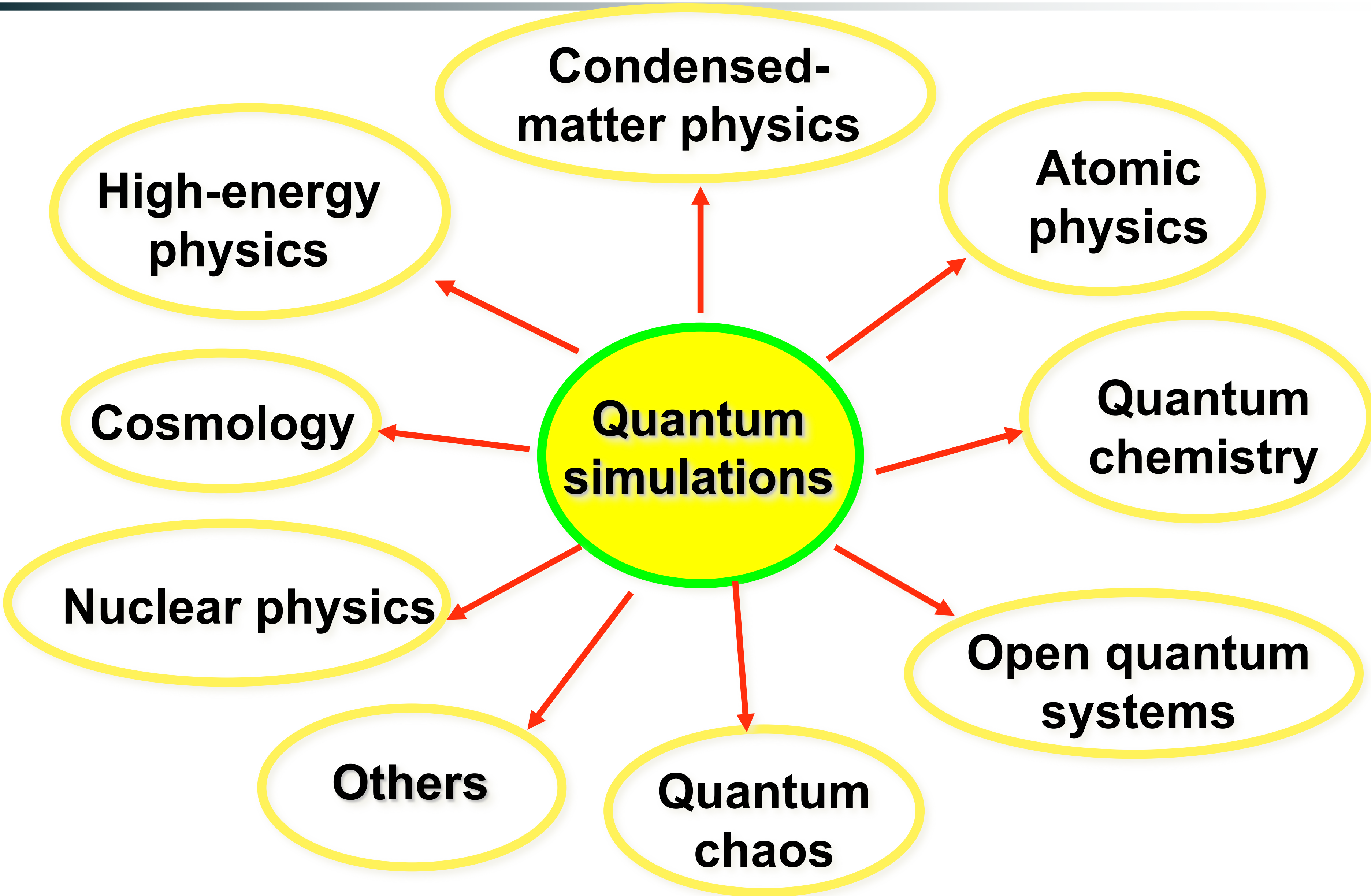
"Simulating Physics with Computers",

***Int. J. Theor. Phys.* 21, 467-488, 1982**

"Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws."

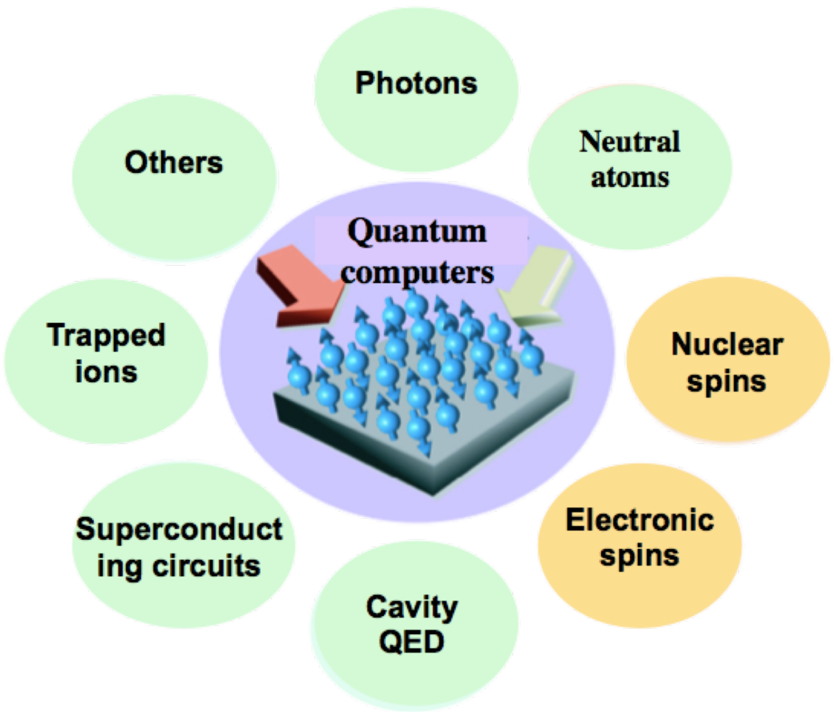


Quantum simulation: Targets



I. M. Georgescu et al., Rev. Mod. Phys., Vol. 86, No. 1, January–March 2014

Quantum simulation: Implementation

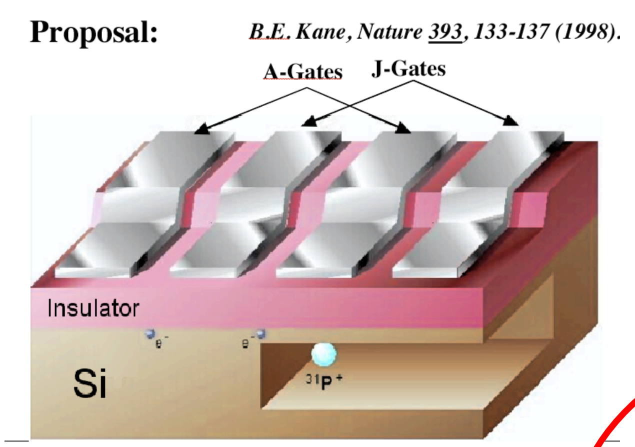


Spin

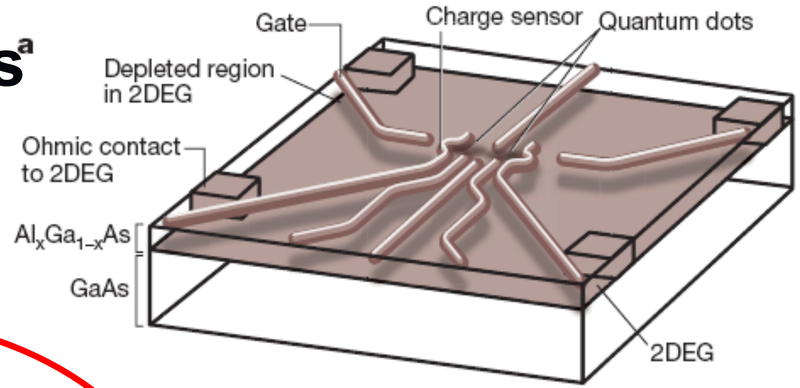
- **Nuclear spins have long decoherence time**
- **Electronic spins have the fast operation time**
- **Spins can be easily manipulated by mature magnetic resonance techniques (NMR, EMR, ODMR, FMR)**

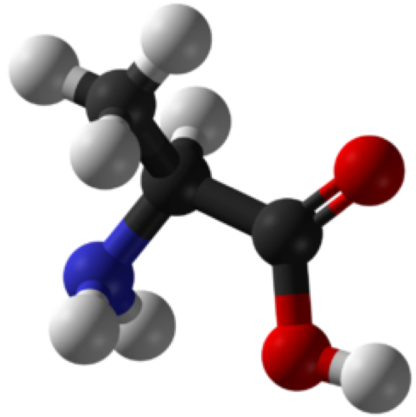
Spin-based QIP is one of most successful physical implementations, and provides inspired technology for others solid systems, as an important testbed for developing quantum control methods.

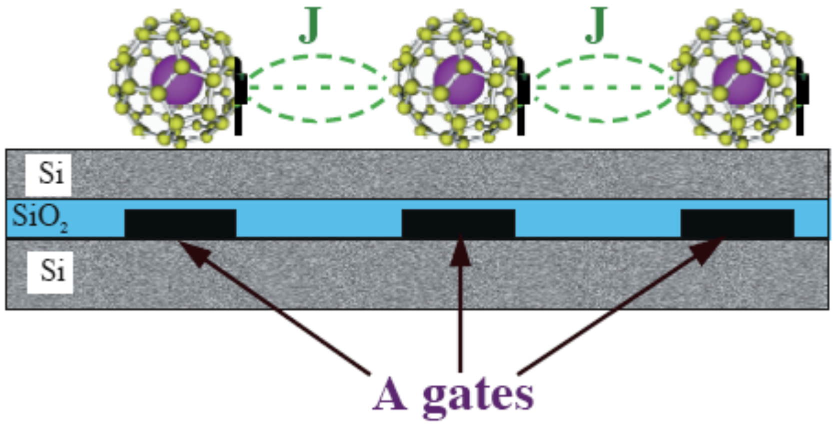
Spin-based quantum information processing

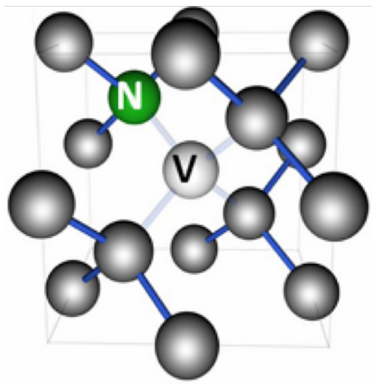


Spintronics[a]





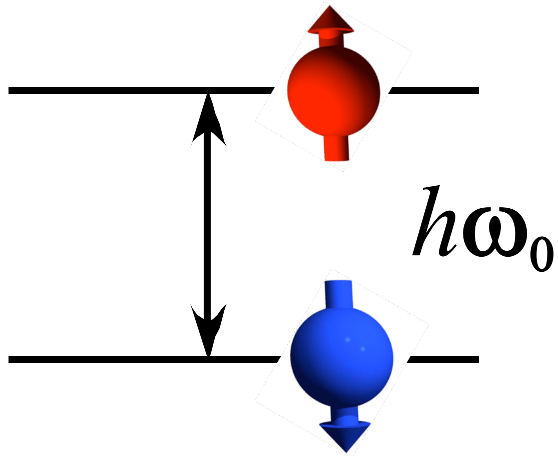




Nuclei + electrons

Quantum control: Spin Magnetic resonance

Spin 1/2 particle in magnetic field



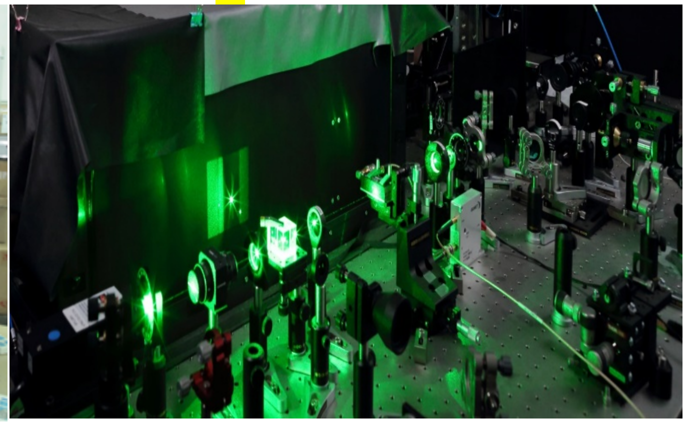
PHz (10^{12}) ODMR

GHz (10^{9}) ESR

MHz (10^{6}) NMR

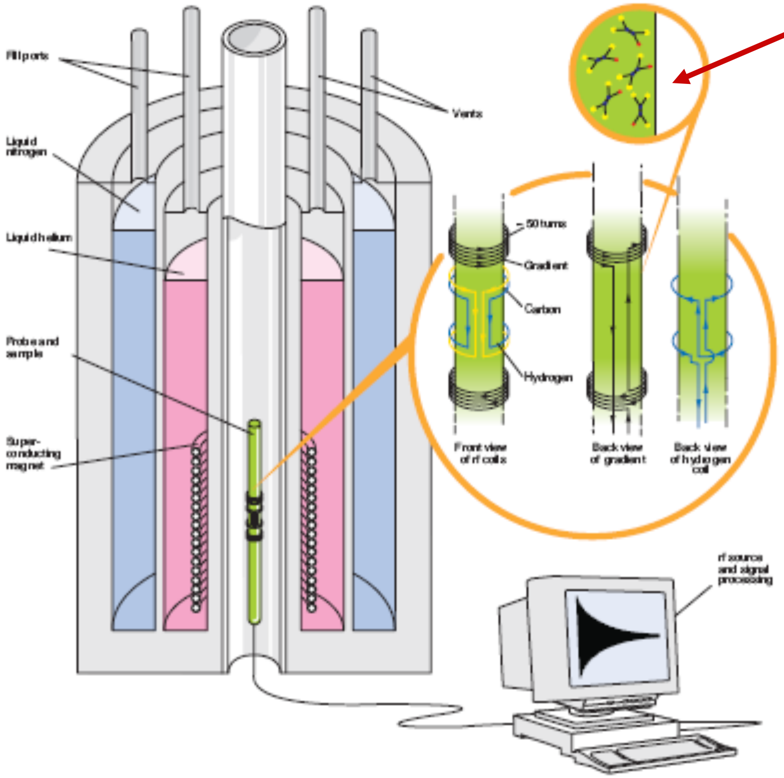
KHz (10^{3}) FMR

Hz (0-10^{2}) LNMR

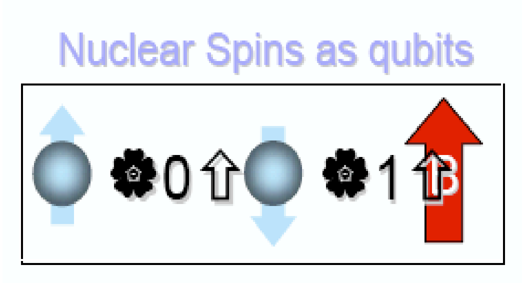


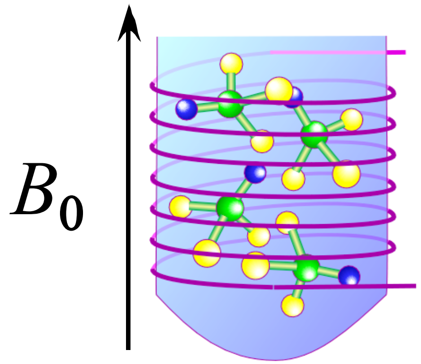
Our platform: NMR QIP

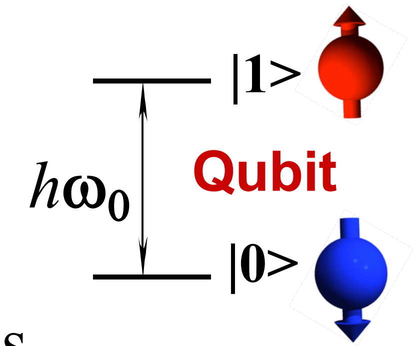
Liquid state NMR **is an excellent system for small quantum registers.**



NMR quantum register







Control：
Radiofrequency pulses
Spin-spin interactions

Initialization： Pseudo-pure state

Readout: Ensemble

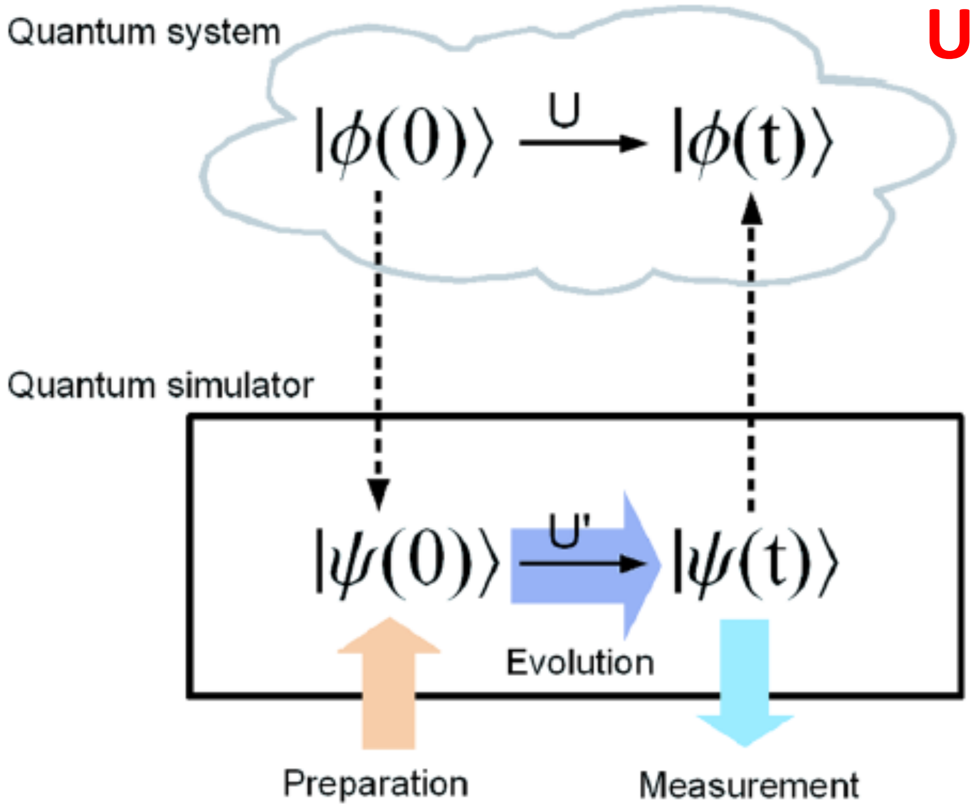
NMR Quantum simulators

Quantum simulator: a controllable quantum system used to simulate or emulate other quantum systems

e.g., many-body interactions, topological orders

$$U = \exp\{-i\hbar H_{\mathrm{sys}} t\}$$

Uncontrollable, unrealistic



$$U' = \exp\{-i\hbar H_{\mathrm{sim}} t\}$$

the mappings

$$|\varphi(0)\rangle \leftrightarrow |\psi(0)\rangle$$

$$|\varphi(t)\rangle \leftrightarrow |\psi(t)\rangle$$

$$U \leftrightarrow U'$$

controllable

NMR systems

I. M. Georgescu et al., Rev. Mod. Phys., Vol. 86, No. 1, January–March 2014

Quantum simulation: How?

Main steps

Mapping ϕ

$|\varphi(0)\rangle \leftrightarrow |\psi(0)\rangle$

$|\varphi(t)\rangle \leftrightarrow |\psi(t)\rangle$

$U \leftrightarrow U'$

Initialization

- Direct state constructio n
- Adiabatic state preparatio n

Hamiltonian engineering

- Lloyd's method
- Average Hamiltonian theory
- Quantum network

Measurement

- State tomography (full information)
- Extract some property: correlation function or spectra of operators

Digital vs. Analog QS

- **Digital QS**
  - State $|\psi\rangle$ to be encoded using the computational basis (qubits)
  - Quantum circuit model: Evolution is implemented through single- and two-qubit gates
  - DQS is universal
  - Example: Quantum chemistry

- **Analog QS**
  - The system and simulator are sufficiently similar
  - Map the evolution of the system to be simulated onto the controlled evolution of the quantum simulator

$$H_{\rm sys} \leftrightarrow H_{\rm sim}$$

  - Robustness
  - Example: Adiabatic quantum simulation (the dynamics or ground state)

DQS: Quantum chemistry

- **First-quantized Hamiltonian:**

$$H = T + V = \frac{p^2}{2m} + V(x)$$

- **Particle's wavefunction (position representation)**

$$|\psi\rangle = \sum_{x=0}^{2^n-1} a_x \, |x\rangle$$

- **Quantum evolution**

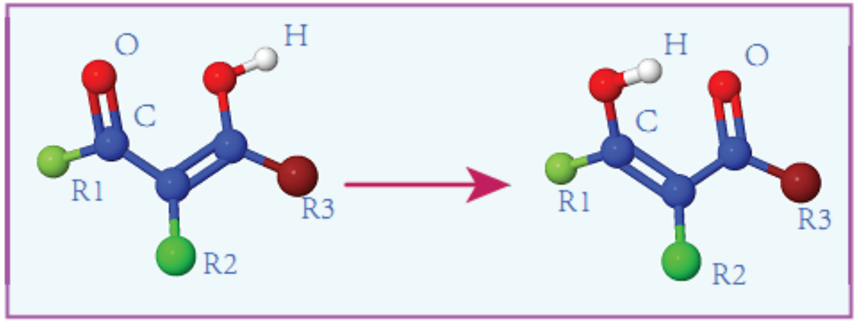
$$U(\delta t) = e^{-i(T+V)\delta t} \approx U_{\mathrm{QFT}} e^{-iT\delta t} U_{\mathrm{QFT}}^{\dagger} e^{-iV\delta t}$$

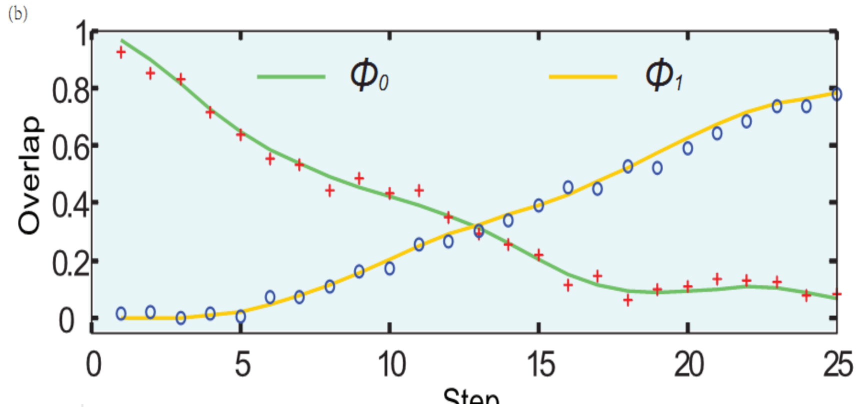
- **Example: chemical dynamics**

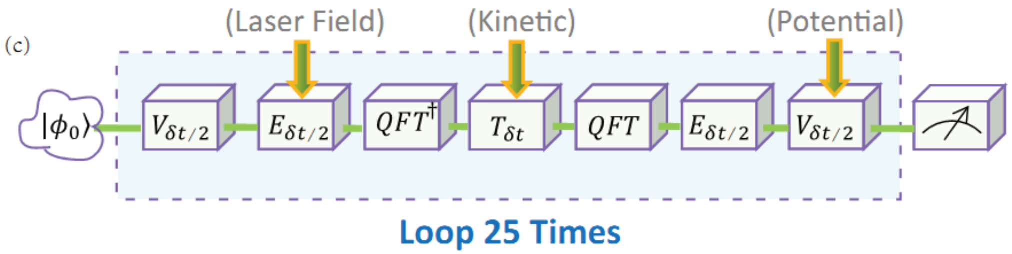
$$H = \sum_i \frac{p_i^2}{2m_i} + \sum_{i<j} \frac{q_i q_j}{r_{ij}}$$

DQS: Simulating chemical reaction

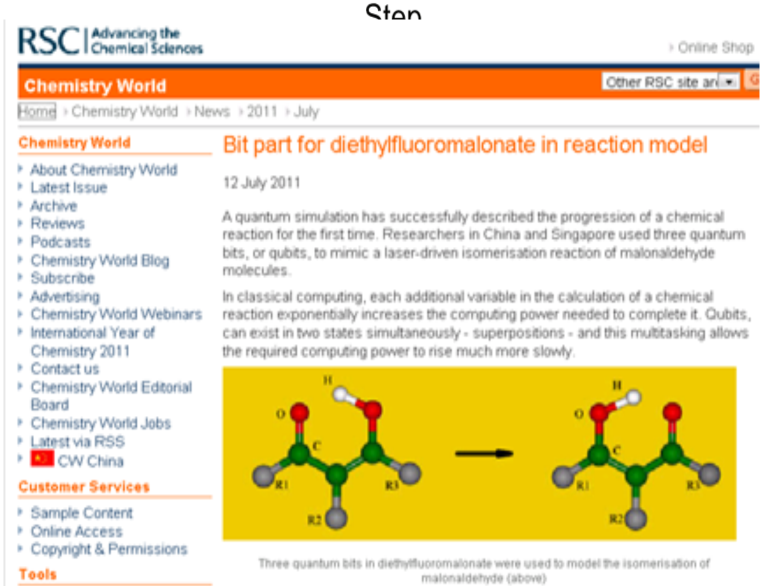
Simulation of Chemical Isomerization Reaction Dynamics







Website report



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Tools

Bit part for diethylfluoromalonate in reaction model

12 July 2011

A quantum simulation has successfully described the progression of a chemical reaction for the first time. Researchers in China and Singapore used three quantum bits, or qubits, to mimic a laser-driven isomerisation reaction of malonaldehyde molecules.

In classical computing, each additional variable in the calculation of a chemical reaction exponentially increases the computing power needed to complete it. Qubits, can exist in two states simultaneously - superpositions - and this multitasking allows the required computing power to rise much more slowly.

Three quantum bits in diethylfluoromalonate were used to model the isomerisation of malonaldehyde (above)

D. Lu et al., **Phys. Rev. Lett.** 107, 020501 (2011)

DQS: Quantum chemistry

- **Second-quantized Hamiltonian:**

$$\hat{H} = \sum_{p,q} h_{pq}\hat{a}_p^+\hat{a}_q + \frac{1}{2}\sum_{p,q,r,s} h_{pqrs}\hat{a}_p^+\hat{a}_q^+\hat{a}_r\hat{a}_s$$

$$[\hat{a}_i, \hat{a}_j^+]_+ = \delta_{ij} \text{ and } [\hat{a}_i, \hat{a}_j]_+ = 0$$

✓ $O(N^4)$ terms, N is the num- ber of single-electron basis functions (i.e. spin-orbitals)

✓ Coefficients are classically evaluated by a preliminary Hartree-Fock procedure

✓ Straightforward mapping to qubits

$$|0\rangle = \text{occupied}, \ |1\rangle = \text{unoccupied}$$

✓ Jordan-Wigner transformation to spins

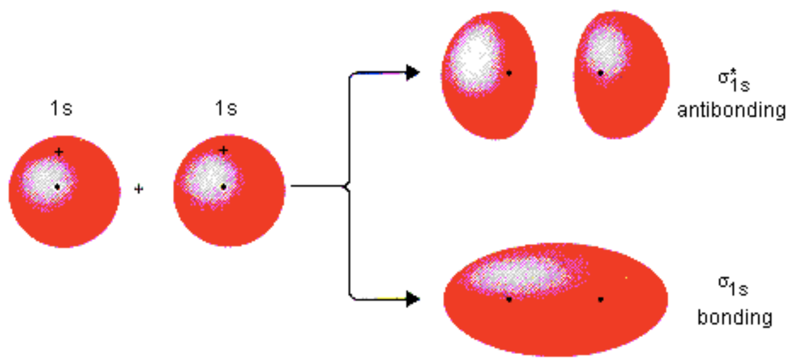
$$\hat{a}_j \rightarrow \mathbf{1}^{\otimes j-1} \otimes \hat{\sigma}^+ \otimes (\hat{\sigma}^z)^{\otimes N-j} \qquad \hat{a}_j^+ \rightarrow \mathbf{1}^{\otimes j-1} \otimes \hat{\sigma}^- \otimes (\hat{\sigma}^z)^{\otimes N-j}$$

DQS: Quantum chemstry

- **Example: MO-LCAO for H_2**

Minimal basis (STO-3G)

Due to symmetry, the Hamiltonian is block-diagonal in this basis



|u> (odd)

|g> (even)

1 qubit for QS

$$\chi^{\uparrow}(\mathbf{x}) = \psi(\mathbf{r})\alpha(\omega)$$

Spin orbital   Spatial orbital   Spin function

$$\chi^{\downarrow}(\mathbf{x}) = \psi(\mathbf{r})\beta(\omega)$$

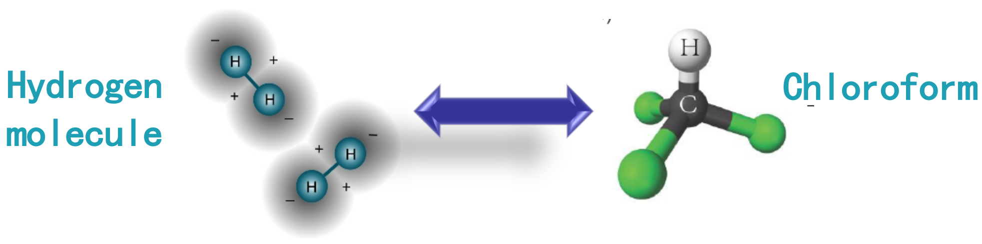
N = 2
6 unique configurations

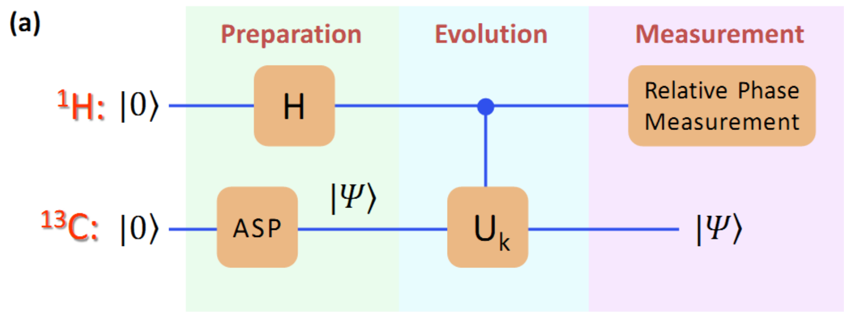
$$\binom{4}{2} = \frac{4!}{2!2!} = 6$$

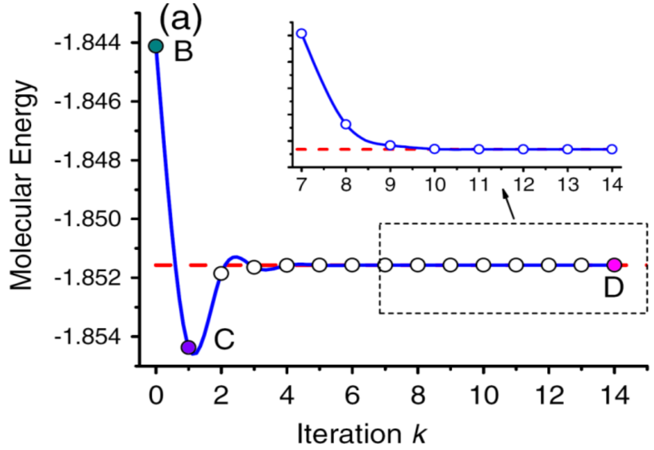
A. Aspuru-Guzik et al., Science, 309, 1704 (2005)

DQS: Simulating hydrogen molecule

We achieve a 45-bit estimation of the ground-sate energy







	Iteration k	Binary value
ϕ_{exp}	0	0.1001000111011001010100001100100000111111110110
	2	0.100100100111010111001011010011000101001001110
	5	0.1001001001110000000011010011101101111011101001
	8	0.100100100111000000001010000111010001000111110
	11	0.10010010011100000000101000011011111001110000000
	14	0.10010010011100000000101000011011111001101010110
ϕ_{th}		0.10010010011100000000101000011011111001101010110

High Lighted Article

J. Du et al., **Phys. Rev. Lett.** 104, 030502 (2010)

AQS: Quantum phase transition

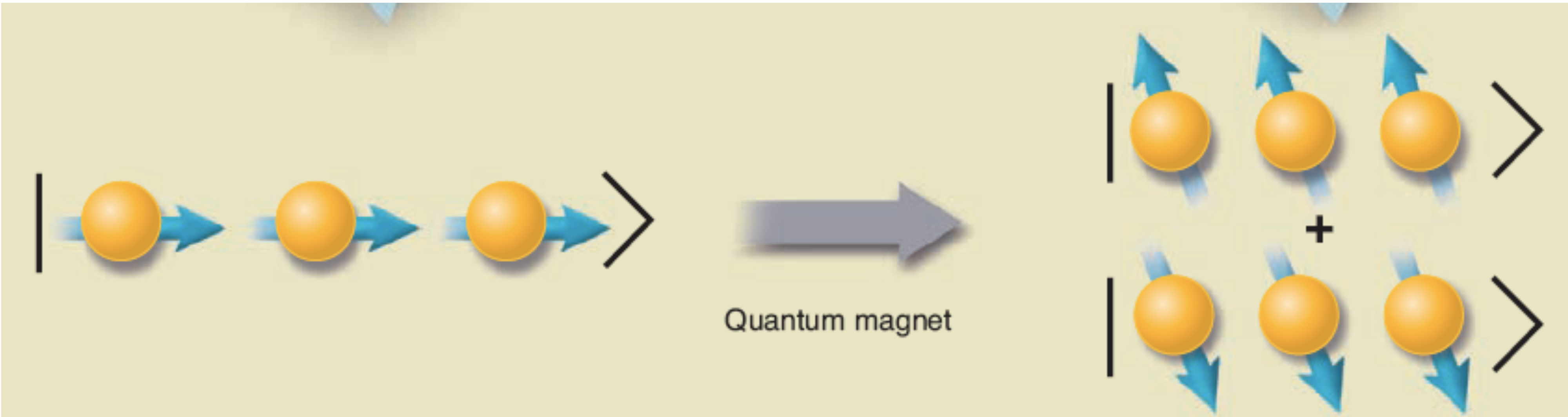
Quantum spin model (Quantum magnets)

$$H=\sum_{i=1}^{n} B_i\sigma_{iz}+\sum_{i<j=1}^{n}\left(J_{ij}^{x}\sigma_{ix}\sigma_{jx}+J_{ij}^{y}\sigma_{iy}\sigma_{jy}+J_{ij}^{z}\sigma_{iz}\sigma_{jz}\right)$$

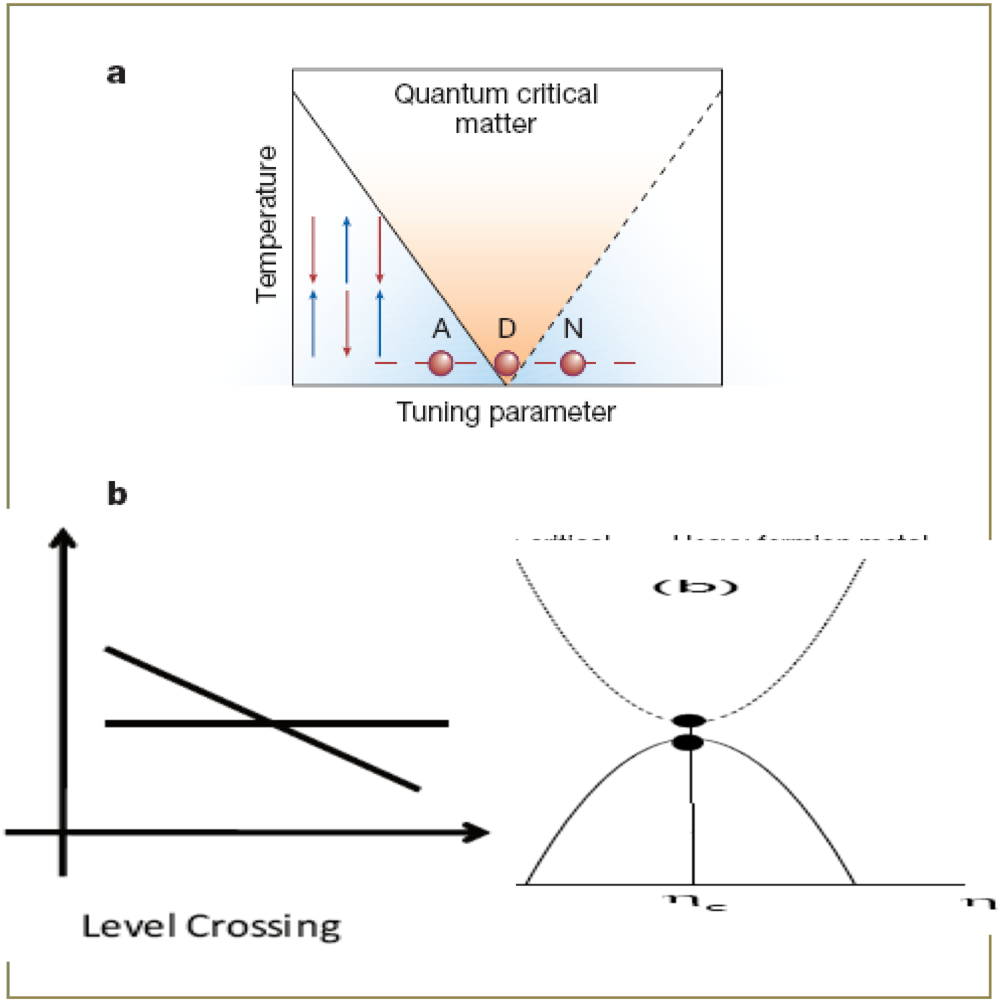
External fields   Heisenberg couplings

Heisenberg isotropic, Ising, XX, XY, XYZ model

Mapping: A more realistic model in that it treats the spins quantum-mechanically, by replacing the spin by a quantum operator (Pauli spin-1/2 matrices at spin 1/2).



Quantum phase transition (QPT)



Non-analyticity of ground level at critical point λ_c

QPT

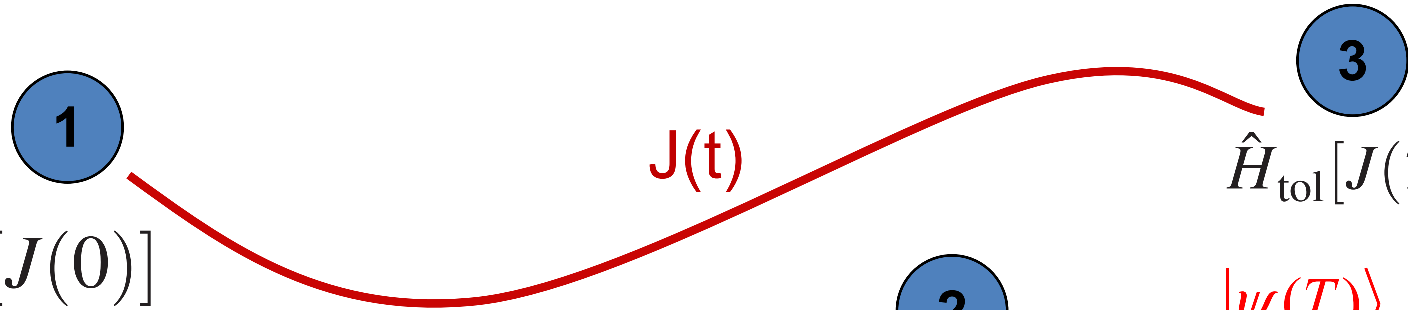
Drastic change of fundamental properties of the ground state (GS), when the parameter λ passes λ_c

Some low lying excited states also change drastically

Adiabatic quantum simulations

Schrödinger equation: $$\frac{d}{dt}|\psi(t)\rangle = -iH(t)|\psi(t)\rangle$$

$$\hat{H}_{tol} = -2J(\hat{\sigma}_1^x\hat{\sigma}_2^y\hat{\sigma}_3^x\hat{\sigma}_4^y + \hat{\sigma}_1^y\hat{\sigma}_2^x\hat{\sigma}_3^y\hat{\sigma}_4^x) - g\sum_i^4 \hat{\sigma}_i^x$$



$\hat{H}_{\mathrm{tol}}[J(0)]$ $\hat{H}_{\mathrm{tol}}[J(T)]$

$|\psi_g(0)\rangle$ $|\psi_g(T)\rangle$

2 Adiabatic evolution

$$T >> \frac{\left\| \langle E_1 | \frac{dH}{ds} | E_0 \rangle \right\|}{g_{\min}^2}$$

One phase Another phase

Linear interpolation $s(t) = \frac{t}{T}$

Adiabatic quantum simulations

Our experiments for many-body systems on QPT:

2005	2009	2014	2014
2 spins →	**3** spins →	**4** spins →	**5** spins
PRA	PRL	PRL	PRL
First exp. Adiabatic QS	Three-body interaction New kind of QPT	Four-body interaction Topological QPT	32-spin QPT Compressed QS

X. Peng et al., Phys. Rev. A 72, 052109 (2005)

X. Peng et al., Phys. Rev. Lett. 103, 140501 (2009)

X. Peng et al., Phys. Rev. Lett. 113, 080404 (2014)

Z. Li, et al., Phys. Rev. Lett. 112, 220501 (2014)

Recent work1: Detecting topological quantum phase transition

Exotic quantum many-body physics

New Physics: Topological orders (new quantum orders)

Kitaev's toric code model

$$H = -\sum_s A_s - \sum_p B_p$$

$$A_s = \sigma^x_{sa}\sigma^x_{sb}\sigma^x_{sc}\sigma^x_{sl}$$

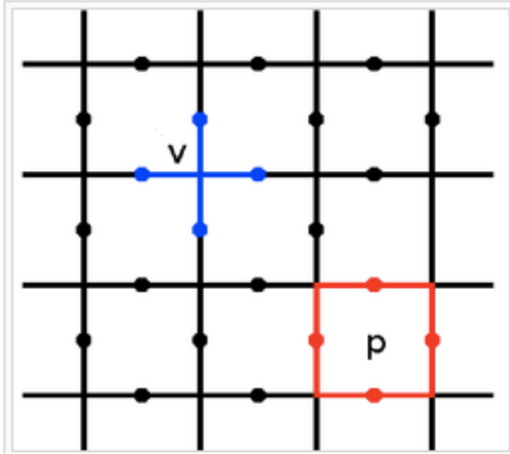
$$B_p = \sigma^z_{ij}\sigma^z_{jk}\sigma^z_{kl}\sigma^z_{li}$$

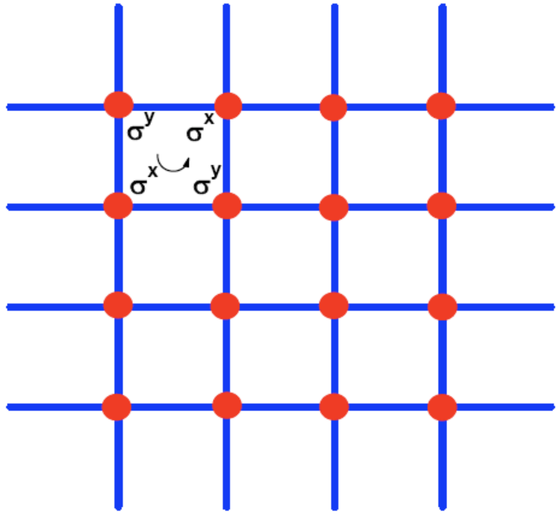
Wen-plaquette model

$$H = -J\sum_i \hat{F}_i,$$

$$\hat{F}_i = \sigma^x_i \sigma^y_{i+\hat{e}_x} \sigma^x_{i+\hat{e}_x+\hat{e}_y} \sigma^y_{i+\hat{e}_y}$$

Z2 topological orders





Topologic orders = pattern of quantum entanglements

A. Kitaev, Ann. Phys. 303, 2 (2003); X. G. Wen, PRL. 90, 016803 (2003)

Experiments?

Great Challenge in experimental study for TOs:

Difficult to realize topological orders directly in real systems

A possible solution: Quantum simulation

Quantum computers provide an alternative way to investigate TOs in experiments (artificial states of matter).

Most previous experiments for the toric-code model:

Photon systems [Nature 482, 489-494 (2012), PRL 102, 030502 (2009), New. J. Phys 11, 083010 (2009)]

NMR systems [PRA 88, 022305 (2013); New J. Phys. 18, 043043 (2016)]

State-based approaches, rather than attempting to realize the Hamiltonian

Hamiltonian Engineering

- **Quantum Control Model**

$$U = e^{-iH_I T} \xleftrightarrow{\overline{H}_p = \phi H_I \phi^{-1}} V_T = e^{-i\overline{H}_p T} = \prod_k e^{-iH_p t_k} V_k$$

- **Lloyd's method** $\hat{H} = \sum_{j=1}^{n} \hat{H}_j$

Trotter-Suzuki formula

$$\hat{U}(t) = e^{-iHt} = (e^{-iH_1(t/m)} e^{-iH_2(t/m)} \dots e^{-iH_k(t/m)})^m + \sum_{i<j} [H_i, H_j] \frac{t^2}{2m} + \dots . \quad ($$

the number of operations $Op_{\text{Lloyd}} \propto t^2 n g^2 / \epsilon$ Polynomial scaling

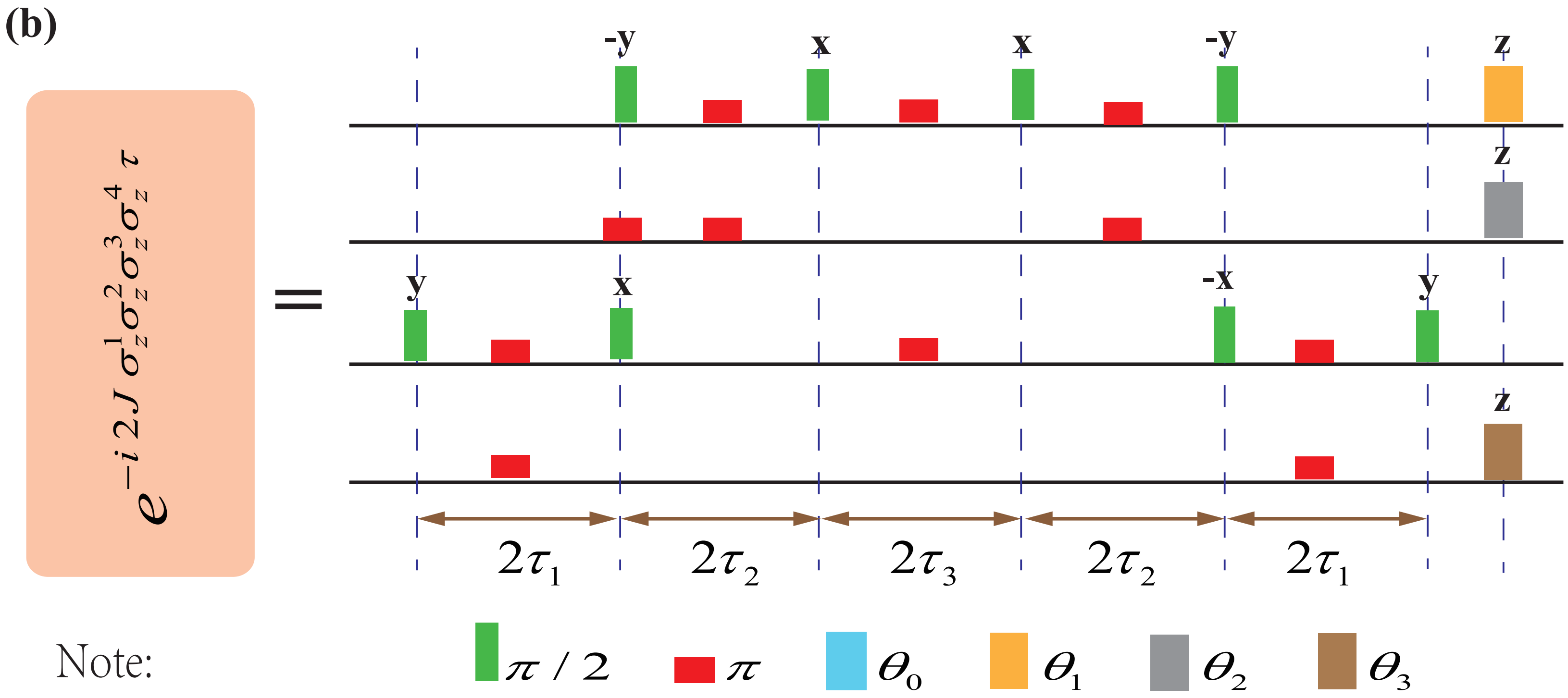
S. Lloyd. Universal Quantum Simulators. Science, 273(5278):1073–1078, 1996.

Hamiltonian Engineering

Many-body interactions

Simulated | Physical

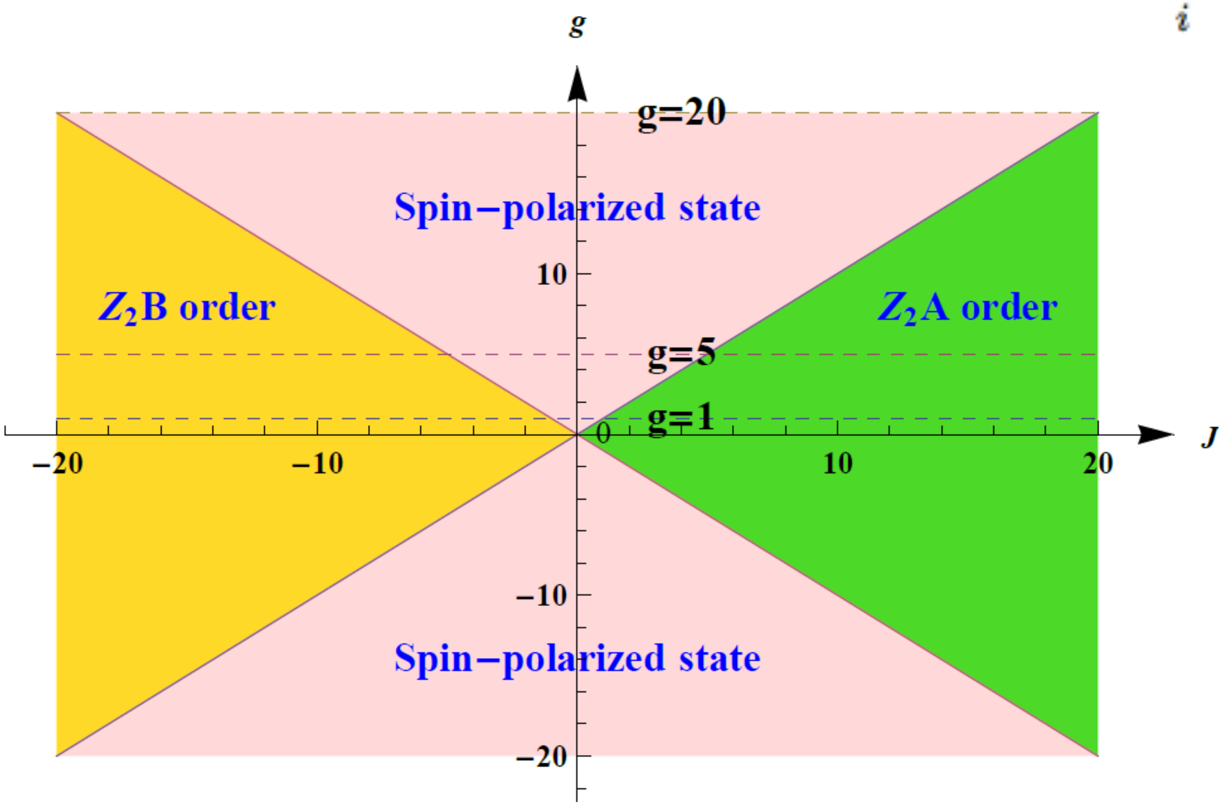
$$\hat{\sigma}_z^1 \hat{\sigma}_z^2 \hat{\sigma}_z^3 \hat{\sigma}_z^4 \longleftarrow H_{nmr} = \sum_{i=1}^{n} \omega_i \sigma_{iz} + \sum_{\substack{i<j=1 \\ i \neq j}}^{n} J_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j$$



Phase diagram

Transverse Wen-plaquette model

$$\hat{H}_{tol} = \hat{H}_{Wen} - g\sum_i \hat{\sigma}_i^x.$$



The transition region depends on the value of |J/g|:

- The transition becomes narrower and sharper as g decreases.
- When g ➔0, Z2B ➔Z2A topological order @ J = 0.

Analogy quantum simulation

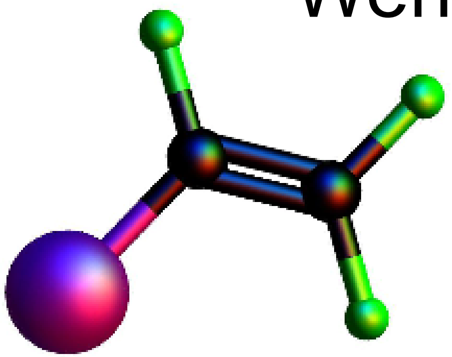
Experiment for TQPT

PRL **113,** 080404 (2014) PHYSICAL REVIEW LETTERS week ending 22 AUGUST 2014

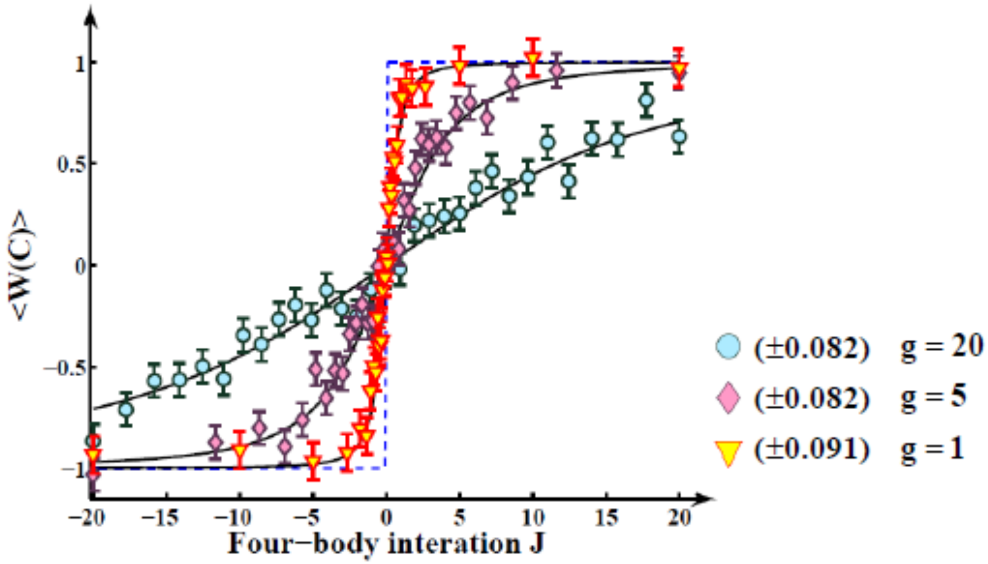
Experimental Implementation of Adiabatic Passage between Different Topological Orders

Xinhua Peng,[1,2,*] Zhihuang Luo,[1] Wenqiang Zheng,[1] Supeng Kou,[3,†] Dieter Suter,[4] and Jiangfeng Du[1,2,‡]

Wen-plaquette model



(a) Wilson loop $\hat{W}(C) = \hat{\sigma}_1^x \hat{\sigma}_2^y \hat{\sigma}_3^x \hat{\sigma}_4^y$



Adiabatic quantum simulation with the background of four-body Wen-plaquette spin model

Non-local order parameter

Recent work2: Identifying Z2 topological order and its breakdown

Characterizing Topological orders

One of the most important questions in condensed matter physics is the description and the classification of different phases of matter.

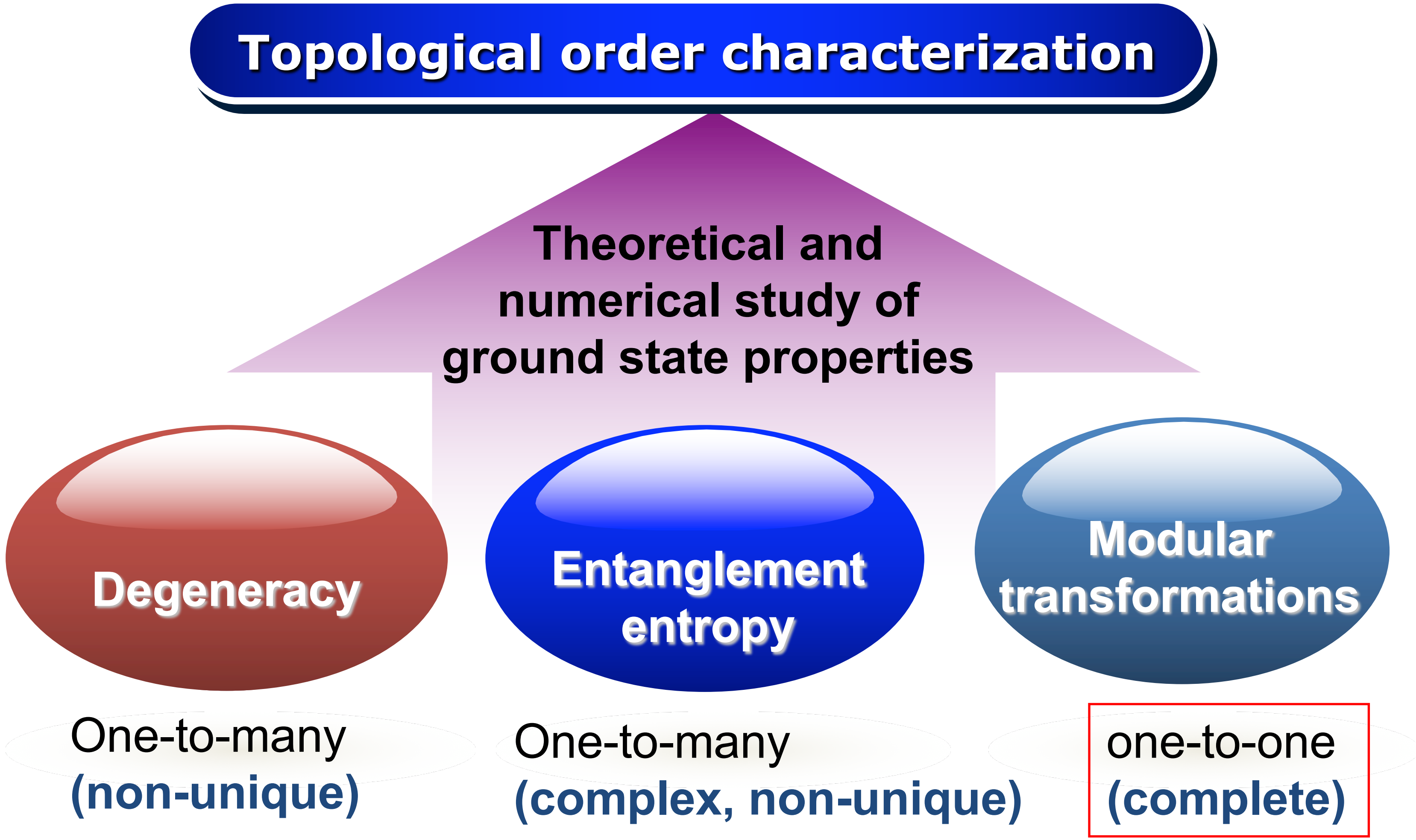
How to classify different topological orders (TOs)?

old question ➔ new objects

Landau's theory	Topologically ordered states
Direct-product states	Quantum many-body states
“short-range entanglement"	“long-range entanglement".
Symmetry	?

an especially challenging task!

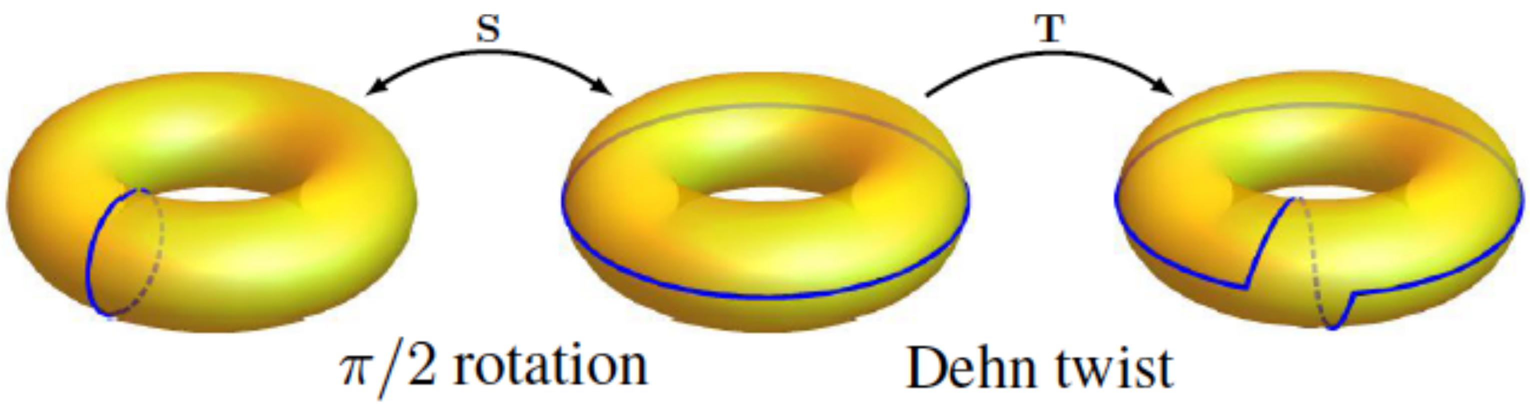
Characterizing topological order



How to identify the topological phase uniquely and purely by experimental means ?

Modular Matrices

- Non-Abelian geometric phases of degenerate ground state
- All the information of quasiparticles statistics and their fusion
- A complete and one-to-one description of TOs
- Non-local order parameter



$$S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$$T = \mathrm{diag}\{1, 1, 1, -1\}$$

braiding and fusion of two anyons
(multual statistics)

self-statistics of all anyons

TOs and modular matrices

Z2 toric code

$$T = \mathrm{Diag}\{1,1,1,-1\}$$

$$S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

Diagonal T matrix: self-statistics of all anyons

S matrix: braiding and fusion of two anyons

Doubled semion

$$T = \mathrm{Diag}\{1, i, -i, 1\}$$

$$S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

Doubled Fibonacci

$$T = \mathrm{Diag}\{1, e^{i\frac{4\pi}{5}}, e^{-i\frac{4\pi}{5}}, 1\}$$

$$S = \frac{1}{\sqrt{5}\varphi}\begin{pmatrix} 1 & \varphi & \varphi & \varphi^2 \\ \varphi & -1 & \varphi^2 & -\varphi \\ \varphi & \varphi^2 & -1 & -\varphi \\ \varphi^2 & -\varphi & -\varphi & 1 \end{pmatrix}$$

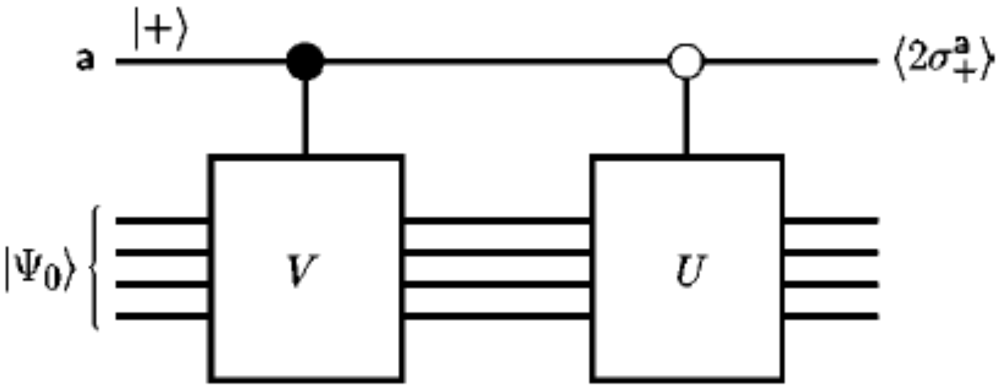
$$\varphi = (1+\sqrt{5})/2$$

Measure modular matrices?

Obtain elements of S/T matrices in standard basis

$$\langle \phi_i^{\text{std}} | S | \phi_j^{\text{std}} \rangle \qquad \langle \phi_i^{\text{std}} | T | \phi_j^{\text{std}} \rangle$$

A solution: Interferometry method



$$2\sigma_+^{\mathsf{a}} = \sigma_x^{\mathsf{a}} + i\sigma_y^{\mathsf{a}}$$

$$\tilde{V} = |0\rangle\langle 0| \otimes \mathbb{1} + |1\rangle\langle 1| \otimes V$$

$$\tilde{U} = |0\rangle\langle 0| \otimes U + |1\rangle\langle 1| \otimes \mathbb{1}.$$

$$\langle 2\sigma_+^{\mathsf{a}} \rangle = \langle \Psi_0 | U^\dagger V | \Psi_0 \rangle$$

$$\boxed{\left|\phi_i^{std}\right\rangle = S^+ U \left|\psi_0\right\rangle, \left|\phi_j^{std}\right\rangle = V\left|\psi_0\right\rangle}$$

R. Somma et al.,Phys. Rev. A, 65, 042323, (2002)

Experimental scheme

Experimental TO characterization

Without a prior state information!

Experimental measurements of modular transformations

Without QST!

Initialization

Random adiabatic evolutions

Generate a set of linearly independent ground states
Hamiltonian-dependent

S, T operations

Permuting the lattice sites (SWAP)

Polynomial gate complexity

measurement and data processing

Interferometry
Gram–Schmidt procedure

Scalable

Efficient quantum network!

Initialization

How to prepare a set of linearly independent states in the degenerated ground-state subspace?

- Random adiabatic method

$$\hat{H}(t) = [1 - s(t)]\hat{H}_{\mathrm{rd}} + s(t)\hat{H}_{\mathrm{T}}^{4}$$

$s(t): 0 \rightarrow 1$ $\quad\quad$ $\hat{H}_{\mathrm{rd}} = \sum_i^4 \sum_{\alpha \in x,y,z} C_i^{\alpha} \hat{\sigma}_i^{\alpha}$

C_i^{α} are randomly generated coefficients between $[-1, 1]$

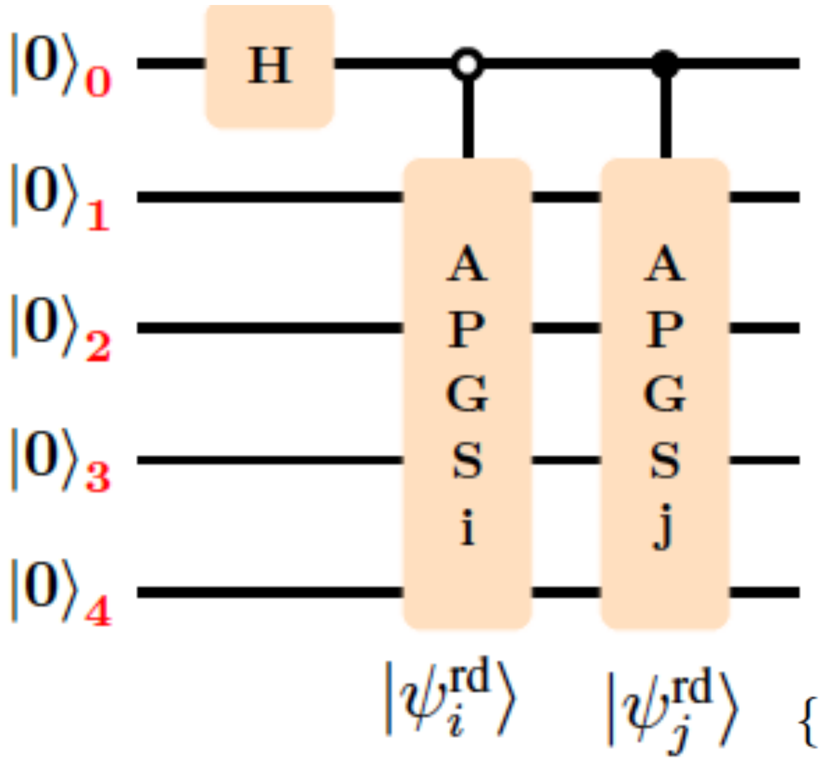
Without the information of string operators

$$\left|\psi^{rd}\right\rangle \rightarrow \sum_{i=1}^{4} c_i \left|\psi_g^i\right\rangle$$

Different $\hat{H}_{\mathrm{rd}}$ $\Longrightarrow$ $\{|\psi_1^{\mathrm{rd}}\rangle, |\psi_2^{\mathrm{rd}}\rangle, \cdots, |\psi_n^{\mathrm{rd}}\rangle\}$

Initialization

- Check $\{|\psi_1^{\text{rd}}\rangle, |\psi_2^{\text{rd}}\rangle, \cdots, |\psi_n^{\text{rd}}\rangle\}$ are linearly independent



$$\langle \hat{\sigma}_0^+ \rangle \mathrm{Tr}(\hat{\rho}_f \hat{\sigma}_0^+) = \frac{1}{2}\langle \psi_i^{\text{rd}} | \psi_j^{\text{rd}} \rangle$$

$$a_i^j$$

$$|\psi_1^{\text{rd}}\rangle = |\phi_1\rangle$$
$$|\psi_2^{\text{rd}}\rangle = a_2^1|\phi_1\rangle + a_2^2|\phi_2\rangle$$

......

$\{|\phi_1\rangle, |\phi_2\rangle, \cdots, |\phi_n\rangle\}$ is an orthogonal basis

If $a_k^k \geq |\xi|$ (ξ is the readout error in the experiment)

$|\psi_k^{\text{rd}}\rangle$ is linearly independent from $|\psi_{k'}^{\text{rd}}\rangle$

S, T operations

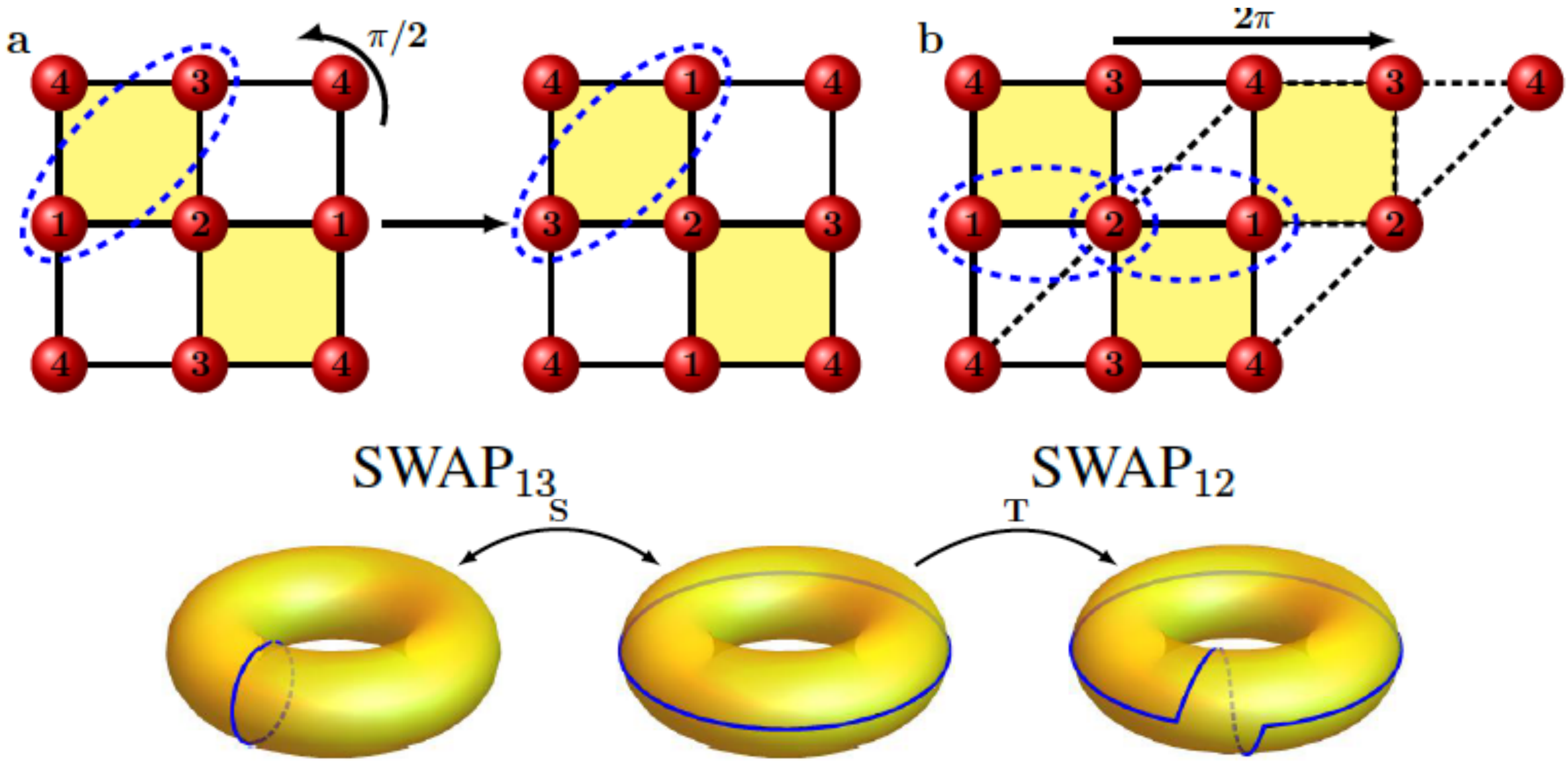
How to physically realize the S, T operations?

Example: A 2x2 torus

Symmetry of the Hamiltonian

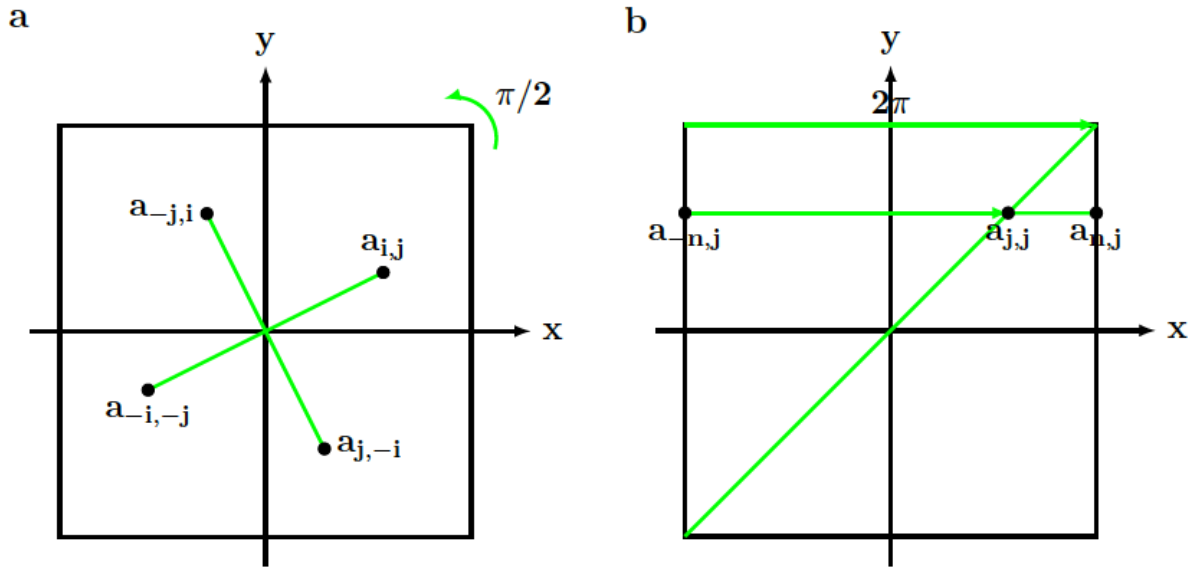
S : 90 rotation

T: Dehn twist



S, T operations

A NxN square lattice of Kitaev toric code model



a series of permutations with length 4 and N

S: $\#(\mathrm{SWAP}) = 3(n^2-1)+3(n-1)+1 = 3n^2+3n+5.$

T: $\#(\mathrm{SWAP}) = N^2 - N - \sum_{j=-n+1}^{n-1} \gcd(n+j, N)$

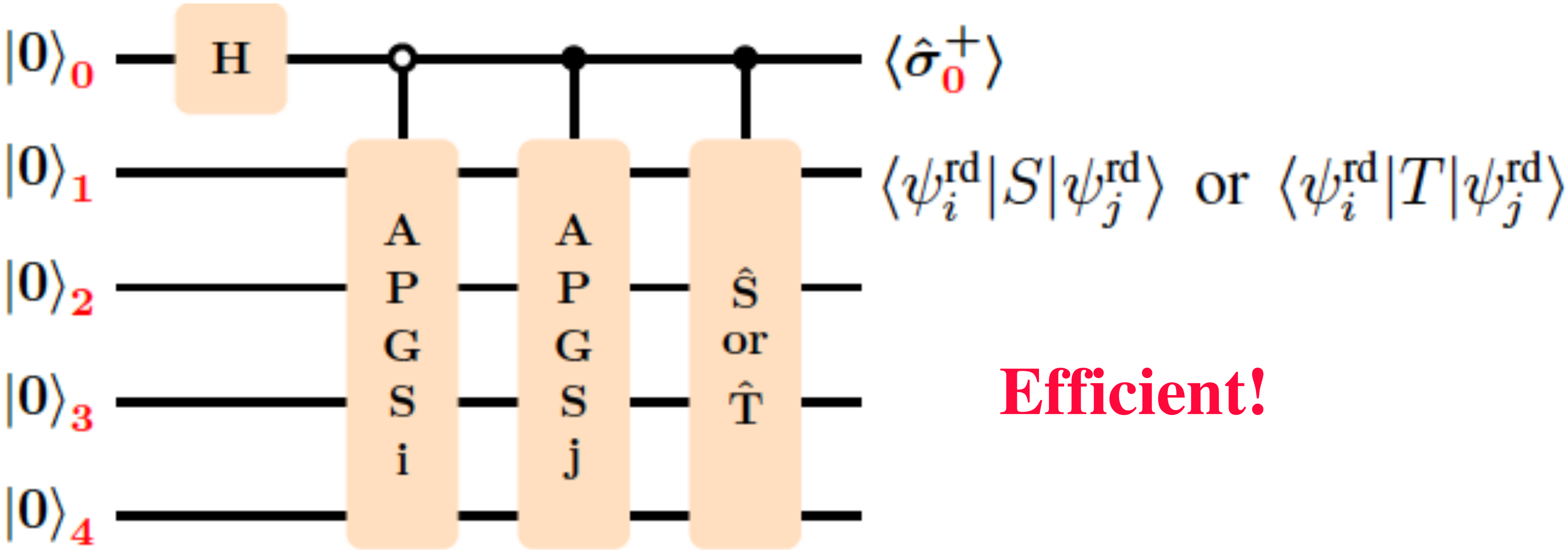
polynomial SWAP gates!

$n = [(N+1)/2]$

Measurement

How to experimentally measure the elements of S, T?

Interferometry method



Efficient!

Note that the S and T matrices obtained in the random basis are not in their standard forms.

Data processing

How to recover the representation of S, T in the standard basis?

- Standard basis: diagonalize the T matrix.

Recovering procedure

$$\langle \psi_i^{\text{rd}} | S | \psi_j^{\text{rd}} \rangle \to \langle \phi_i | S | \phi_j \rangle \to \langle \phi_i^{\text{std}} | S | \phi_j^{\text{std}} \rangle$$

random basis   orthogonal basis   standard basis

$$\langle \psi_i^{\text{rd}} | T | \psi_j^{\text{rd}} \rangle \to \langle \phi_i | T | \phi_j \rangle \to \langle \phi_i^{\text{std}} | T | \phi_j^{\text{std}} \rangle$$

Constructing orthogonal basis

$$\langle \psi_i^{\text{rd}} | \mathcal{O} | \psi_j^{\text{rd}} \rangle = \sum_k \underbrace{A_{ik}}_{\text{Measured}} \langle \phi_k | \mathcal{O} | \phi_j \rangle \Longrightarrow \langle \phi_i | \mathcal{O} | \phi_j \rangle$$

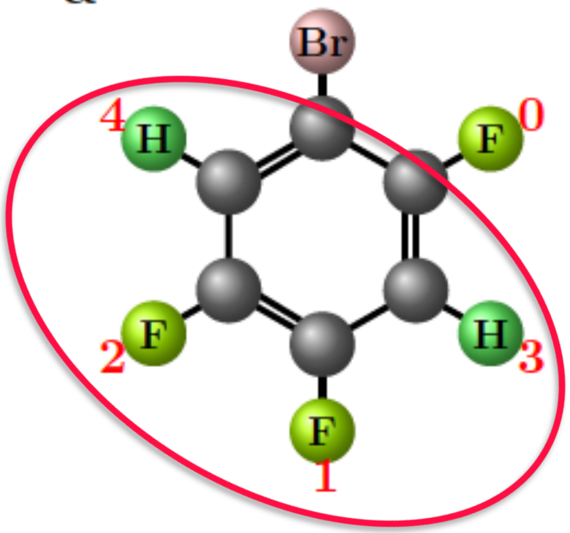
Recovering standard basis: Optimize, diagonalize and transform

$$T_{ab} = \exp(\mathrm{i}\theta_a)\delta_{ab} \qquad S_{1a} = 1$$

NMR Quantum simulator

1-bromo-2,4,5-trifluorobenzene

a



b

	0	1	2	3	4	T_2^* (ms)	T_1 (s)
0	-38032					40	0.8
1	258.4	-45908				40	0.8
2	107.3	-7.5	-48358			40	0.8
3	1454.0	1184.1	51.7	2712		100	1.5
4	51.2	101.1	1430.3	-4.5	2736	100	1.5

Physical system
$$H_{\mathrm{NMR}} = \sum_{j=1}^{5} \pi \nu_j \sigma_j^z + \sum_{1 \le j < k \le 5} \frac{\pi}{2}(J_{jk} + 2D_{jk})\sigma_j^z \sigma_k^z$$

F1+H1+H2+H3

Kitaev toric code model
$$\hat{H}_{Z_2}^4 = -2(\sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x + \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z)$$

Step 1. State preparation

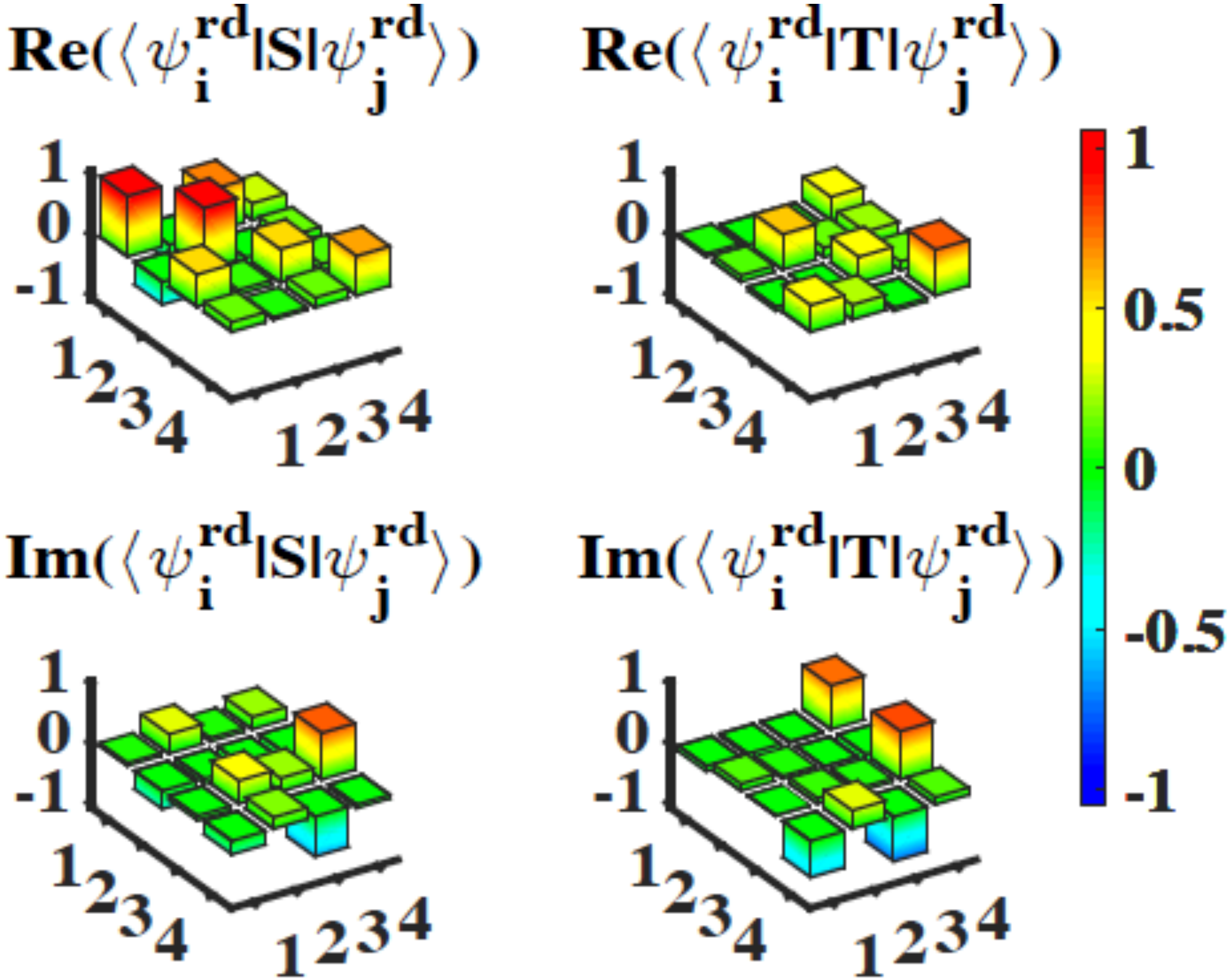
Experimental measure
h = 0

$\langle\psi_1^{\mathrm{rd}}\|\psi_2^{\mathrm{rd}}\rangle$	$-0.3344+0.4432i$
$\langle\psi_1^{\mathrm{rd}}\|\psi_3^{\mathrm{rd}}\rangle$	$0.6932-0.1028i$
$\langle\psi_1^{\mathrm{rd}}\|\psi_4^{\mathrm{rd}}\rangle$	$0.1882+0.1908i$
$\langle\psi_2^{\mathrm{rd}}\|\psi_3^{\mathrm{rd}}\rangle$	$-0.2362-0.4322i$
$\langle\psi_2^{\mathrm{rd}}\|\psi_4^{\mathrm{rd}}\rangle$	$0.1532-0.0482i$
$\langle\psi_3^{\mathrm{rd}}\|\psi_4^{\mathrm{rd}}\rangle$	$-0.0942+0.4642i$

a_2^1	$-0.3344+0.4432i$
a_2^2	$0.8317+0.0000i$
a_3^1	$0.6932-0.1028i$
a_3^2	$0.0495-0.1916i$
a_3^3	$0.6854+0.0000i$
a_4^1	$0.1882+0.1908i$
a_4^2	$0.1582+0.1190i$
a_4^3	$-0.2773+0.4033i$
a_4^4	$0.8059+0.0000i$

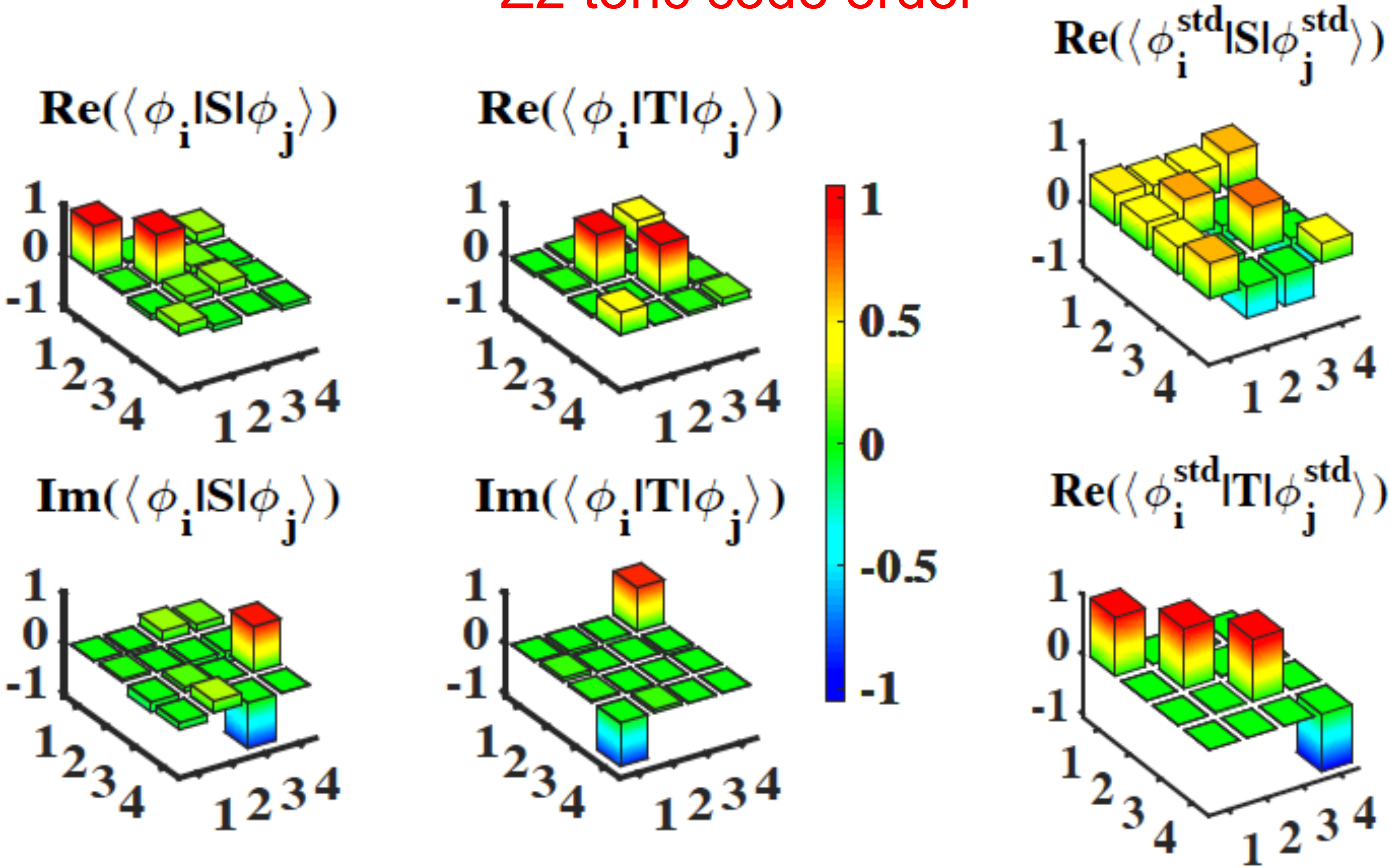
Step 2. Measuring S/T matrices

Experimental S, T matrices in randomly generated linearly independent ground states (h = 0)



Step 3. Recovering standard S/T

Z2 toric code order



What's more: Robustness

- A pertured model: Detuning and disordered Hamiltonian:

$$\hat{H}_{\mathrm{T}} = \hat{H}_{\mathbb{Z}_2} - h\sum_i \hat{\sigma}_i^z - \sum_i \epsilon_i\, \hat{\sigma}_i^z,$$

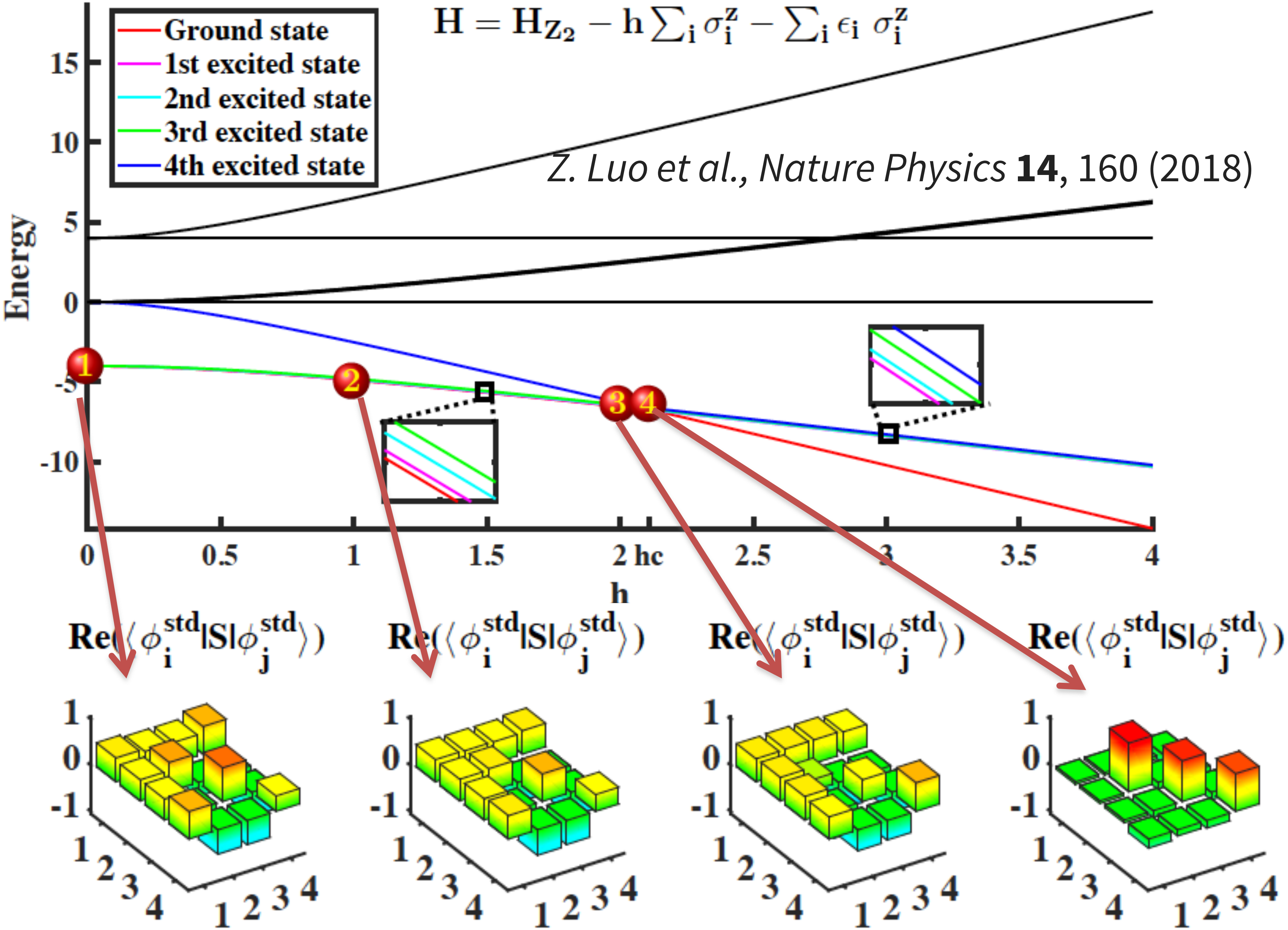
detuning **disordered**

- Detuning the system from the Kitaev soluble point

  A small enough but finite detuning

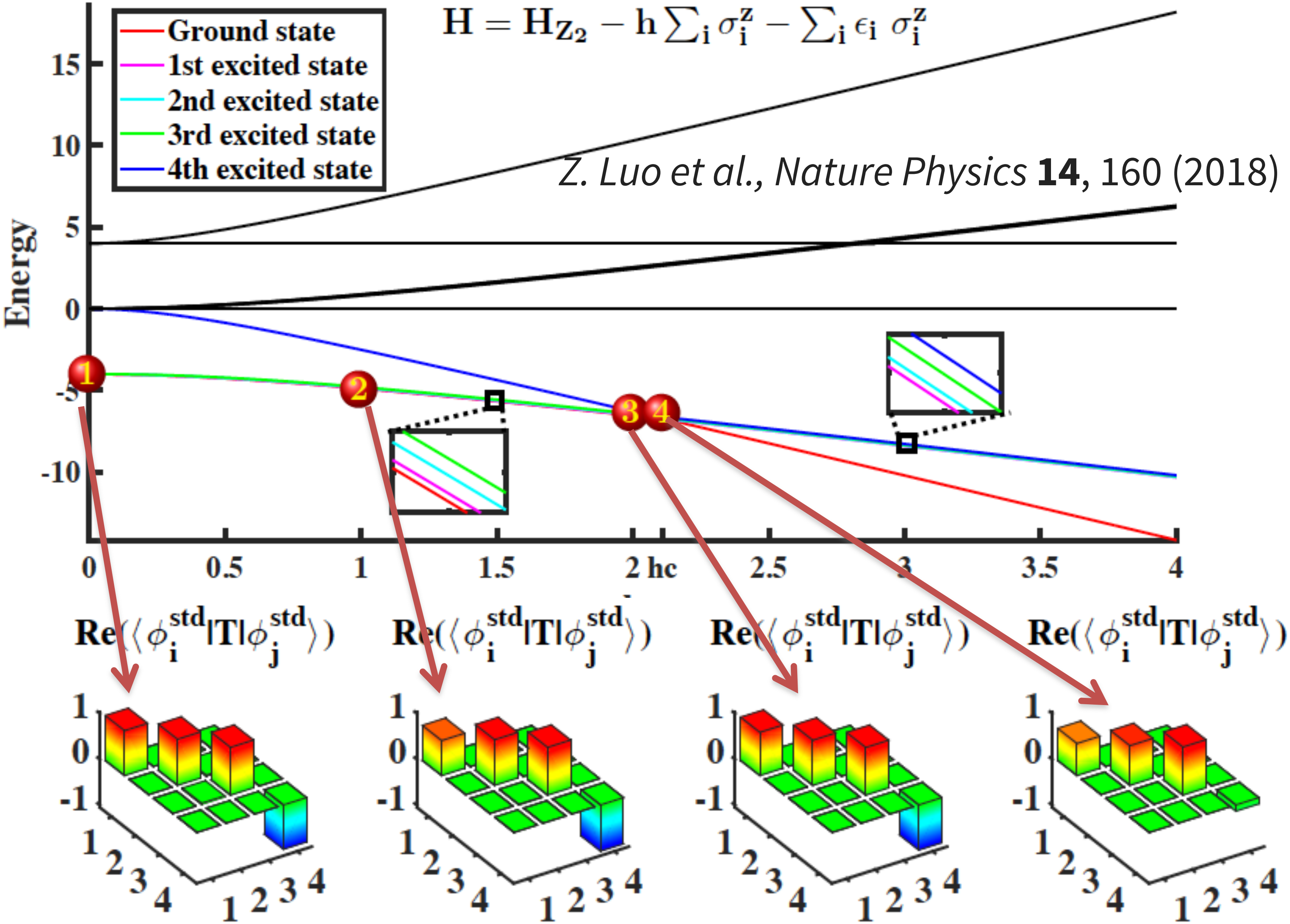
  - preserve the topological nature of the phase while introducing a finite correlation length
  - Phase transition: robustness of this approach / topological phase
- A small inhomogeneity: break all accidental translation symmetries to mimic more realistic situations

Phase transition



Z. Luo et al., Nature Physics **14**, 160 (2018)

Phase transition



Z. Luo et al., Nature Physics **14**, 160 (2018)

Recent work3: Measuring out-of-time correlation

Scrambling is intimately related to chaos

Information not lost, but scrambled

Out-of-Time-Ordered Correlator

$$\langle W^\dagger(t)V^\dagger(0)W(t)V(0)\rangle$$

Spread over many-body degrees of freedom (entanglement)

Black holes scrambles quantum information as soon as possible.
Fastest scrambler in nature

$$\lambda_L = \frac{2\pi}{\beta}$$

Out-of-Time-Order Correlation

Normal correlation (accessible)

$$\langle \hat{W}^\dagger(t)\hat{W}(t)\hat{V}^\dagger(0)\hat{V}(0)\rangle_\beta$$

Out-of-time-order correlation (OTOC)

$$\langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0)\rangle_\beta$$

$$\hat{W}(t) = e^{i\hat{H}t}\hat{W}e^{-i\hat{H}t}$$

H: an interacting many-body Hamiltonian
W(0) and V(0): two commuting operators

$$\mathrm{Re}[F(\tau)] = 1 - \langle |[\hat{W}(\tau),\ \hat{V}]|^2\rangle/2$$

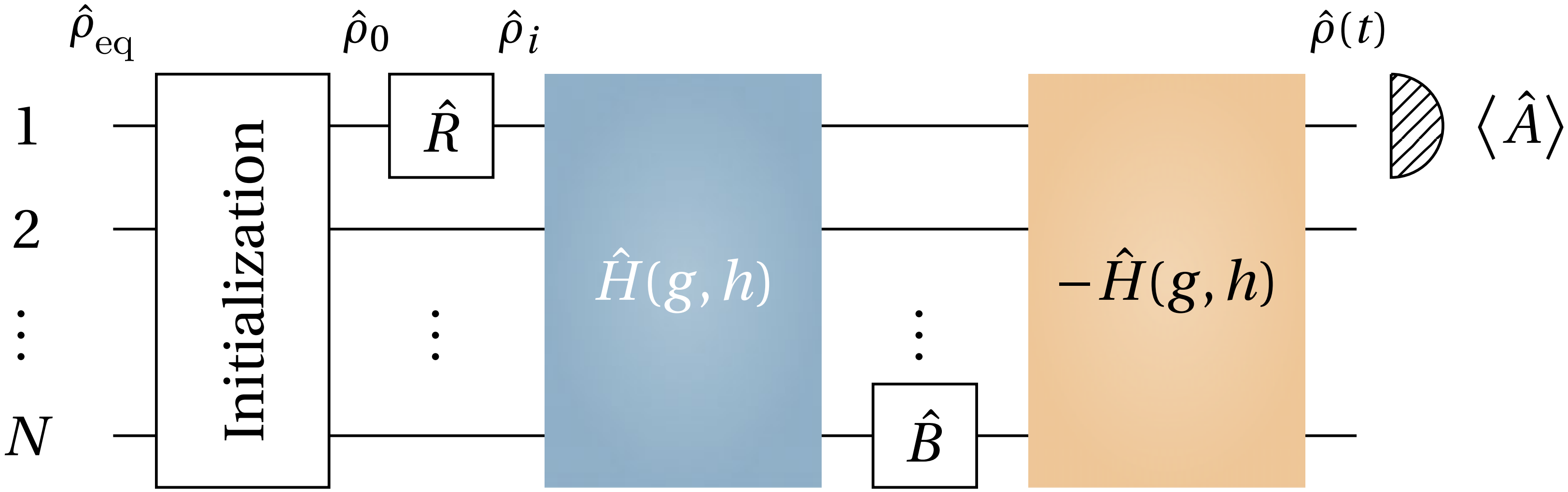
`scrambling' of quantum information

inaccessible to any reasonable local measurement

Experimental scheme

- **Prepare an initial density matrix** $\hat{\rho}_i \propto \hat{A} = \hat{\sigma}_1^\alpha, \alpha = x, y \text{ or } z$
- **Simulate a unitary evolution** $\hat{U}(t) = e^{i\hat{H}t}\hat{B}e^{-i\hat{H}t}$
- **Measure $\hat{A}$ under $\rho(t)$** $\langle \hat{O} \rangle_{\beta=0} = \mathrm{Tr}\left(\hat{U}(t)\hat{\rho}_0\hat{U}^\dagger(t)\hat{A}\right)$

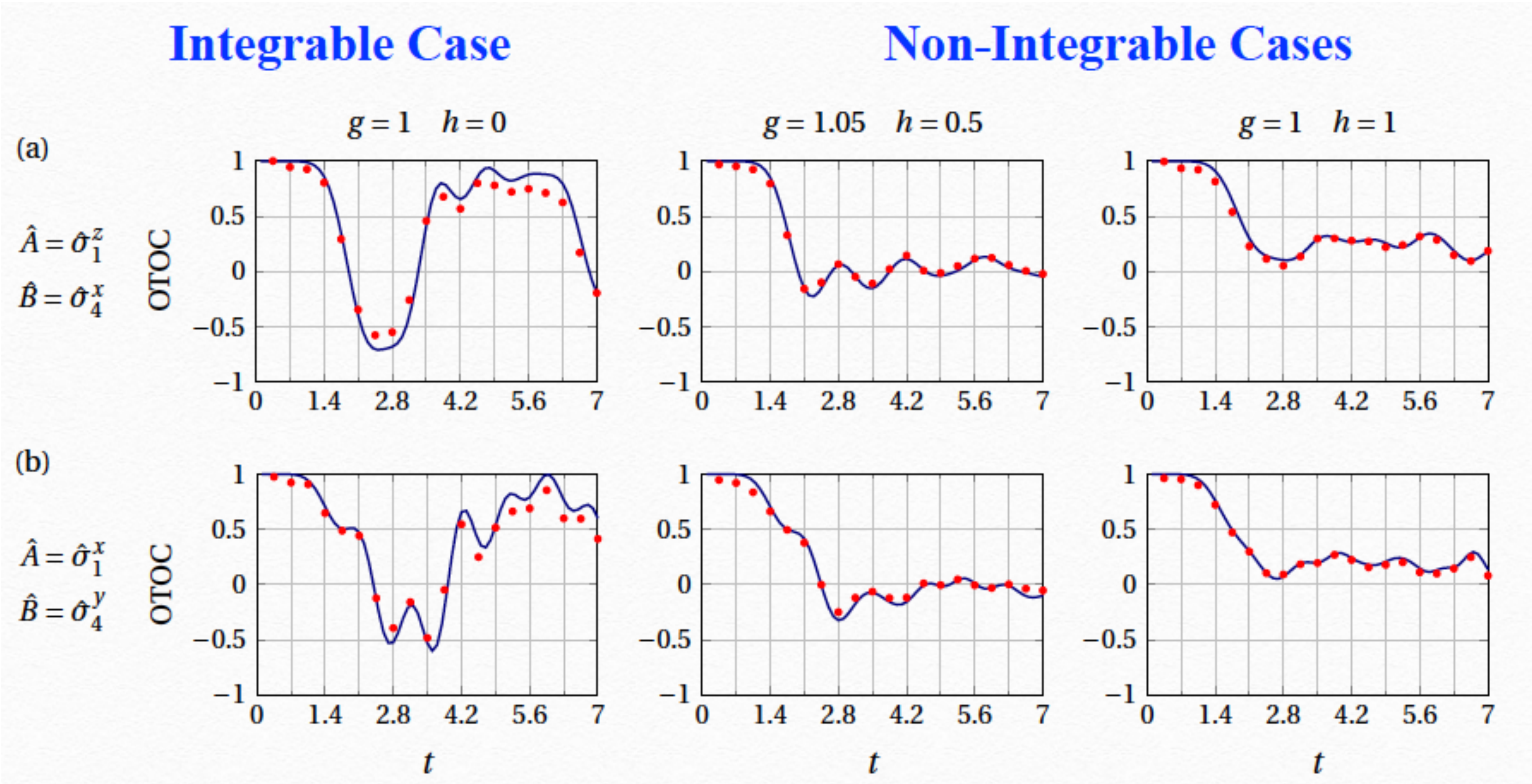
$\hat{\rho}_{\mathrm{eq}} = \sum_{i=1}^4 \epsilon_i \hat{\sigma}_i^z \quad \hat{\rho}_0 = \sigma_1^z$



$\hat{R} = \mathbf{1},\ \hat{R}_x(-\pi/2),\ \hat{R}_y(\pi/2)$ for $\hat{A} = \hat{\sigma}_1^z,\ \hat{\sigma}_1^y,\ \hat{\sigma}_1^x$

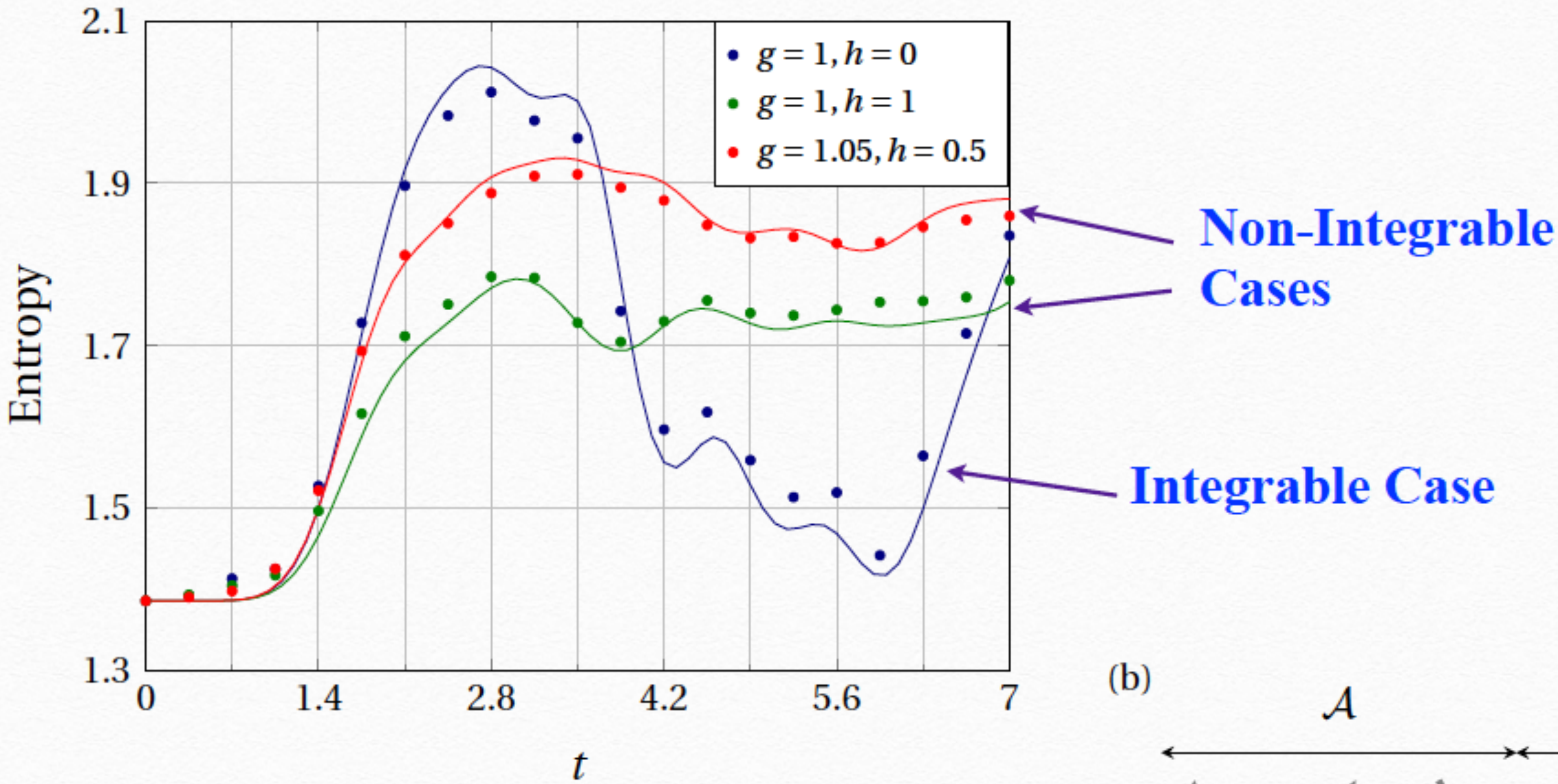
$\hat{B} = \hat{\sigma}_N^\gamma$ a local unitary operator

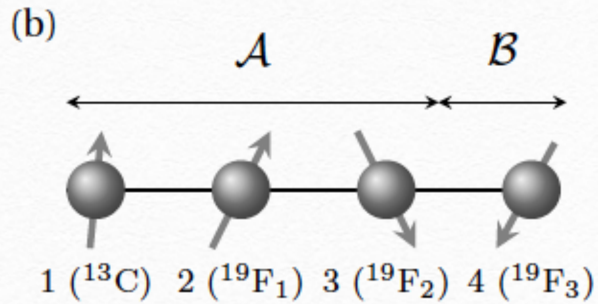
Experimental results



Experimental results

$$\exp(-S_A^{(2)}) = \sum_{M \in B} \mathrm{Tr}[\hat{M}(t)\hat{V}(0)\hat{M}(t)\hat{V}(0)]$$



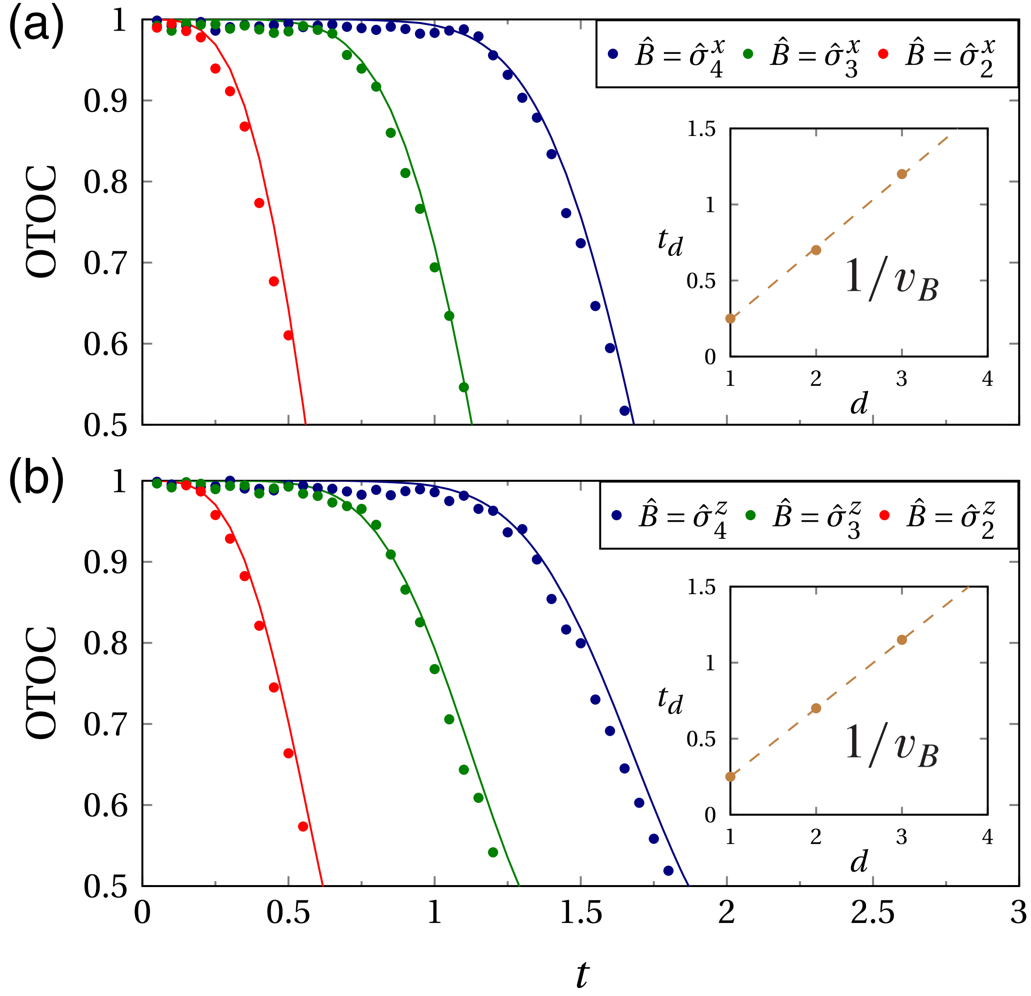


Integrable Case: Information Oscillates

Non-Integrable Case: Information Scrambles

Experimental results

Measurement of butterfly velocity v_B



It quantifies the speed of a local operator growth in time and defines a light cone for chaos, which is also related to the Lieb-Robinson bound

The OTOC also provides a tool to determine the speed for correlation propagating

$$F(t) = a - be^{\lambda_L(t-|x|/v_B)} + \cdots$$

$|x|$: distance between two operators

Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW X **7,** 031011 (2017)

Measuring Out-of-Time-Order Correlators on a Nuclear Magnetic Resonance Quantum Simulator

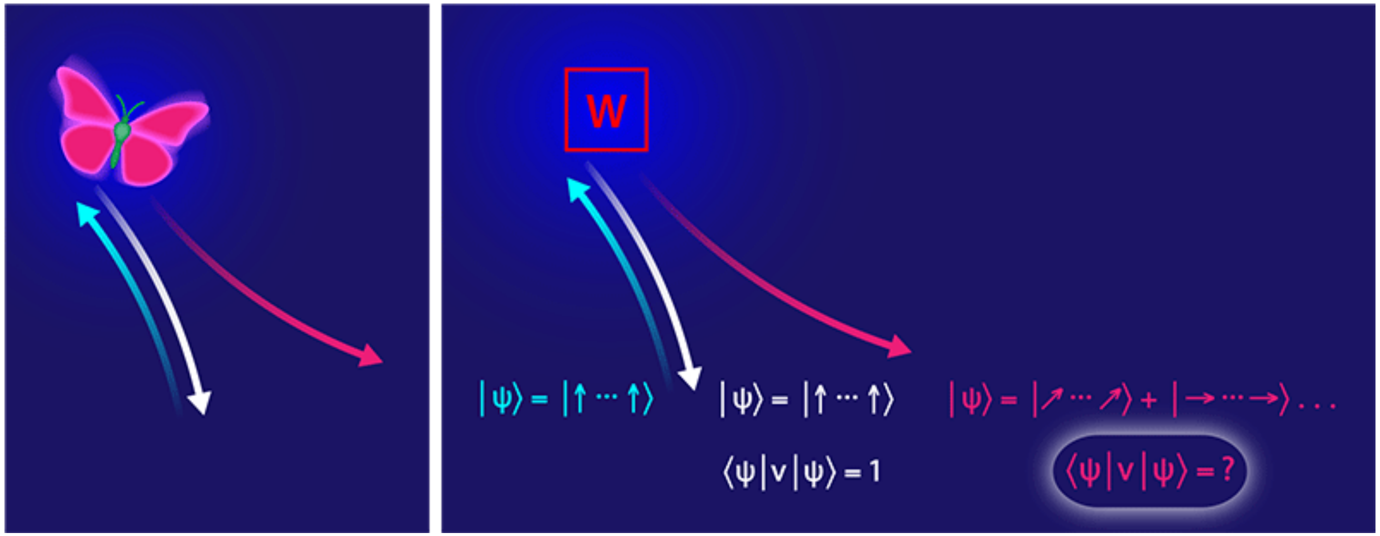
Jun Li,[1] Ruihua Fan,[2,3] Hengyan Wang,[3] Bingtian Ye,[3] Bei Zeng,[4,5,2,*] Hui Zhai,[2,6,†] Xinhua Peng,[7,8,9,‡] and Jiangfeng Du[7,8]

Physics

VIEWPOINT

Seeing Scrambled Spins

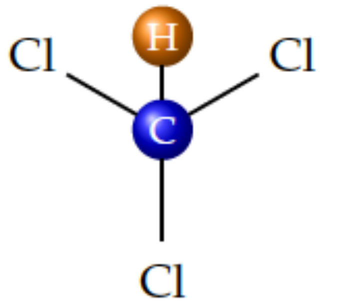
Two experimental groups have taken a step towards observing the “scrambling” of information that occurs as a many-body quantum system thermalizes.

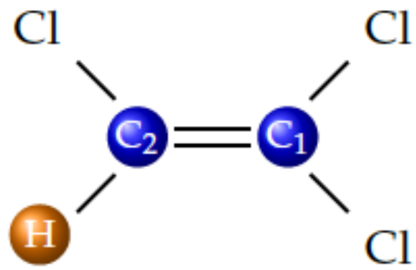


Recent work4: Controlling quantum dynamics based on quantum simulator

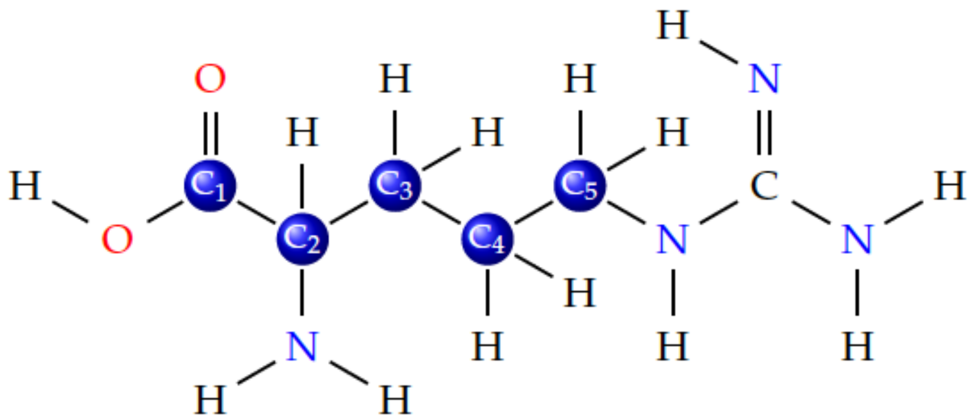
Control of Spin systems

- Closed quantum systems





$$\mathrm{H}(t) = H_d + \sum_i u_i(t) H_i$$



Transformation of quantum state

$$\frac{d\hat{\psi}(t)}{dt} = -iH\hat{\psi}(t)$$

$$\hat{\psi}(t) = U(t)\hat{\psi}(0)$$

Generation of Unitary Operators

$$\frac{d}{dt}U(t) = -iHU(t)$$

H is Hermitian, $H^\dagger = H$

U is Unitary, $U^\dagger U = I$

Control of Spin systems

- Open quantum systems

Lindblad equation

$$\dot{\rho} = -i[H_S + H_C(t), \rho] + \underbrace{\sum_j \gamma_j \left[L_j \rho L_j^\dagger - \frac{1}{2}\left(L_j^\dagger L_j \rho + \rho L_j^\dagger L_j \right) \right]}_{\mathcal{R}\rho\text{:relaxation part}}$$

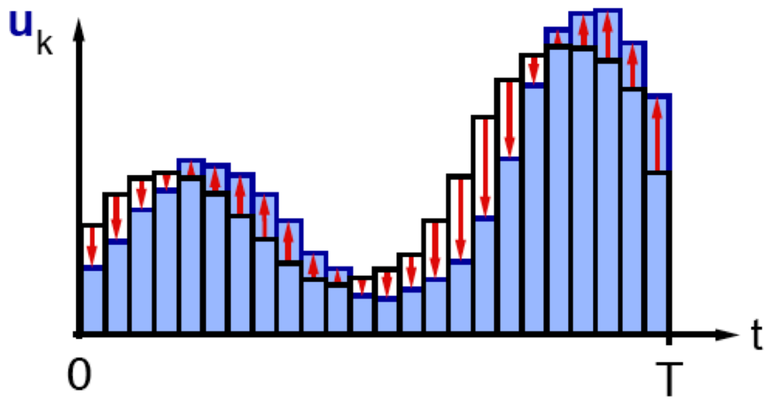
where $\{L_j\}$: Lindblad generators, γ_j: relaxation parameters.

N^2-1, non-negative

More complex, uncontrollable

Control of Spin systems

GRAPE (Gradient ascent pulse engineering)



$U(0) = \mathbf{1}$ $\qquad$ $P(T) = U_F$

desired propagator: U_F

performance: $\left| \left\langle U_F \middle| U(T) \right\rangle \right|^2$

Classically calculated

Gradients

$$\boxed{u_k(t) \longrightarrow u_k(t) + \varepsilon\, Re \left\{ \left\langle P(t) \middle| -i\, H_k U(t) \right\rangle \left\langle U(t) \middle| P(t) \right\rangle \right\}}$$

Khaneja et a., J. Magn. Reson. 172, 296-305 (2005)

Control of Spin systems

Two Key Challenges

Complexity: Exponential growth of Hilbert space

n spin-1/2 system: **2^n x 2^n**

Noise: irreversibly affects control performance

a) Operator errors

b) Relaxation effect from the environment

Difficult part: Performance (fitness), Gradients

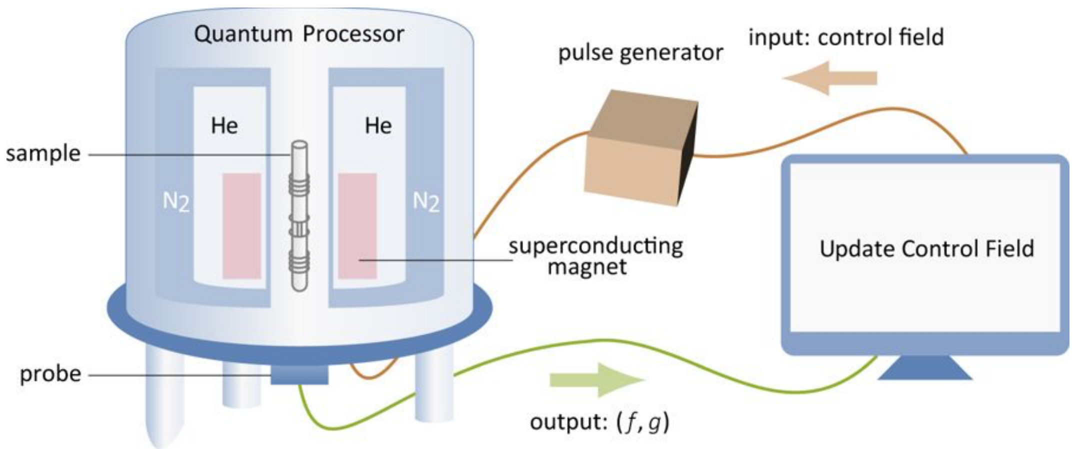
Can they be calculated on quantum simulator?

Control of Spin systems

PRL **118,** 150503 (2017) PHYSICAL REVIEW LETTERS week ending 14 APRIL 2017

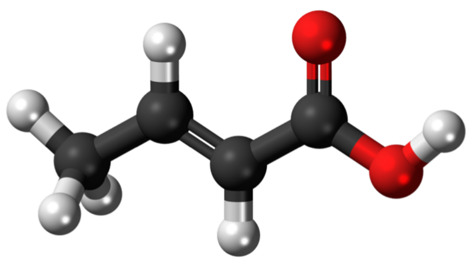
Hybrid Quantum-Classical Approach to Quantum Optimal Control

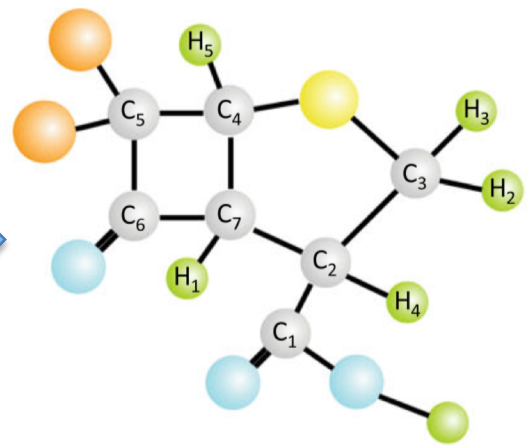
Jun Li,[1,*] Xiaodong Yang,[2] Xinhua Peng,[2,3,†] and Chang-Pu Sun[1]



- Experimental measurements of gradients and the cost function on a quantum simulator
- Experimental close-loop method, using classical computer to update the parameters

7 qubits



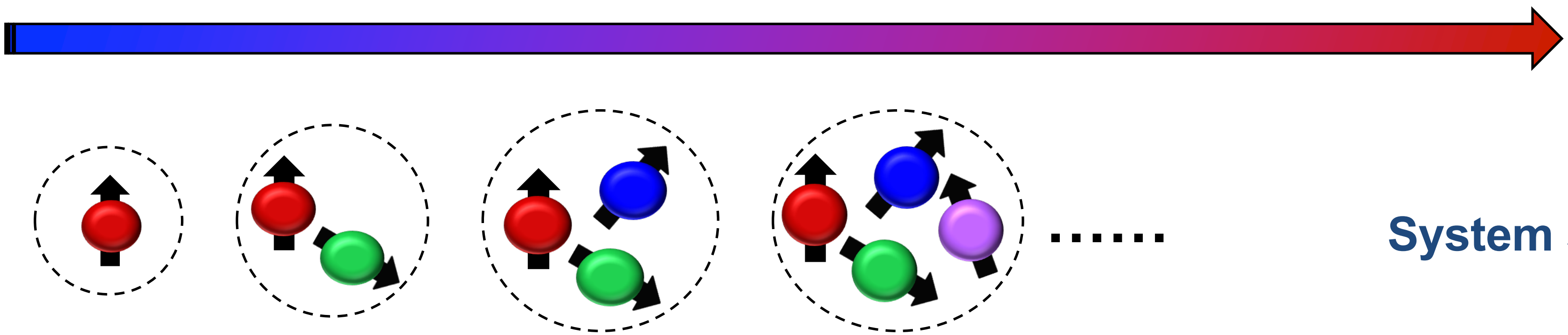


12 qubits

Laflamme group, npj Quantum Information (2017)

Conclusion and Outlook

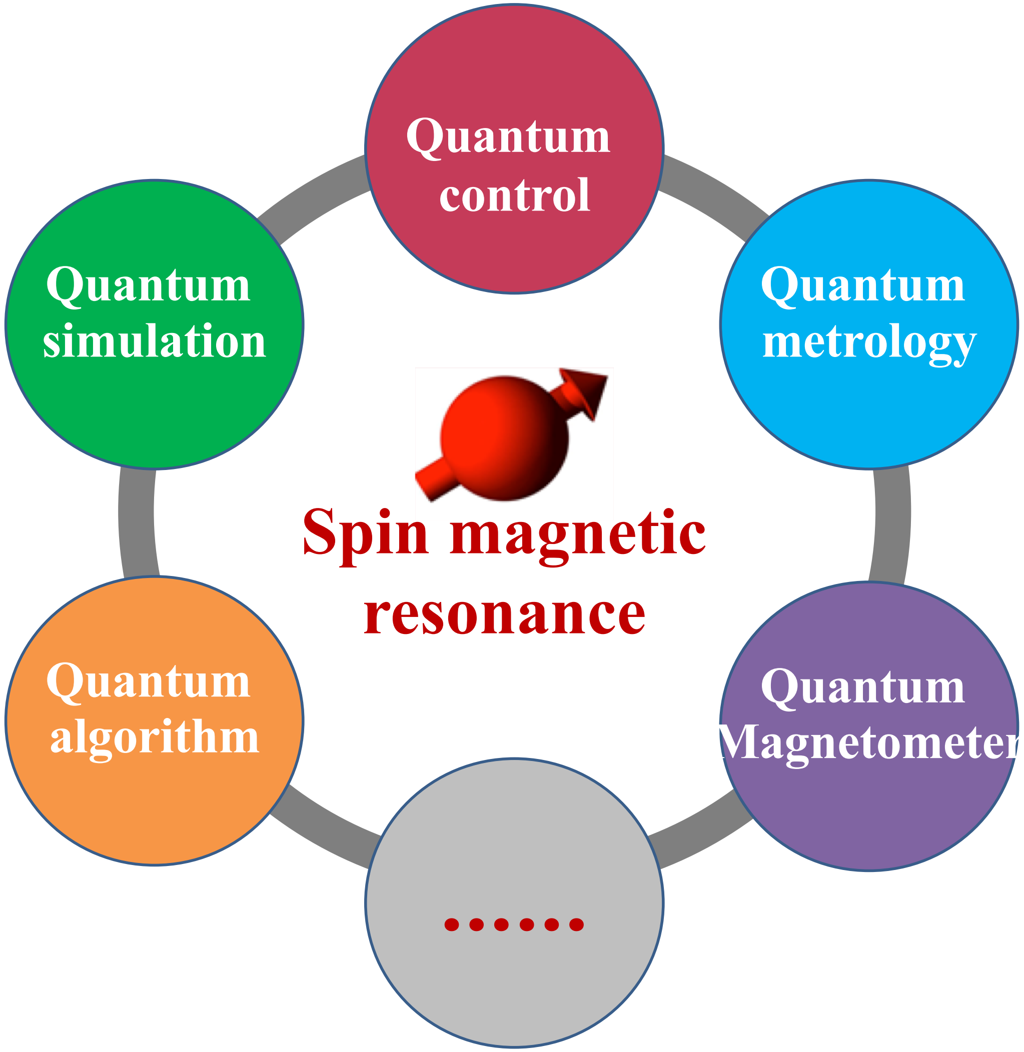
Liquid state NMR is an excellent system for small quantum registers, and one can see the extraordinary achievement of NMR QIP from testing quantum theory to quantum information processing.



System size

Quasi-static Commercial spectrometer (high field)	Dynamical Home-built spectrometer (low field)	Experimental Methods
Independent	Correlated	Environment

Outlook



- Spin is among the most promising physical systems for quantum control.
- Spin holds the promise of realizing various novel quantum applications.
- NMR is a good testbed for QIP.

Thanks to ……

Collaborators:
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Ji Bian
Xi Chen
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Wenjie Xu
…….

Thanks for your attention!

CAS Key Laboratory of Microscale Magnetic Resonance
(Directed by Prof. Jiangfeng Du)

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